

JOURNAL OF RESEARCH AND PRACTICE FOR ADULT LITERACY, SECONDARY, AND BASIC EDUCATION

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RESEARCH

Evaluating Number Sense in Community College Developmental Math Students

By Dorothea A. Steinke

The Case for Measuring Adults' Numeracy Practices

By Diana Coben and Anne Alkema

PRACTITIONER PERSPECTIVE

Problem Posing and Problem Solving in a Math Teacher's Circle

By Eric Appleton, Solange Farina, Tyler Holzer, Usha Kotelawala, and Mark Trushkowsky

Stepping Over the Line: Applying the Theories of Adult Learning in a GED Math Class

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Standards and Professional Development

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FORUM: THE CHALLENGES OF ADULT NUMERACY

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Where to Focus so Students Become College and Career Ready?

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Time Well Spent: Making Choices and Setting Priorities

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RESOURCE REVIEW

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By David J. Rosen



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THE COABE JOURNAL, CELEBRATING 40 YEARS AS A MAJOR VOICE IN ADULT EDUCATION

Dear Readers,

We are pleased and excited to present the Spring issue of the journal. In a departure for this journal, this is a special issue devoted to numeracy. In particular, we would like to extend our gratitude to Dr. Lynda Ginsburg of Rutgers University, who served as the guest editor for this issue. While all of the articles in this issue (except, of course, the regular columns and the Forum pieces) went through the normal review process, Dr. Ginsburg worked extremely hard to recruit the authors and to provide abundant feedback to all of them. She worked tirelessly on this; truly, we could not have produced this special issue without her.

In her research article, Dorothea Steinke identifies some of the problems with developmental math and ways to improve student outcomes in these community college courses. It is of interest because the instrument used to identify students' lack of knowledge of specific numeracy components which she used was practitioner developed and could have important applications on many campuses. Diana Coben and Anne Alkema, both from New Zealand, discuss their efforts to develop a numeracy practices measure and they place the need for such a measure within the broader literature on numeracy for adults.

This issue includes three brief practitioner articles. The first article by Eric Appleton, Solange Farina, Tyler Holzer, Usha Kotelawala, and Mark Trushkowsky describes a professional development approach employed by community college instructors in New York City. It is designed by the participants, allowing them to find answers to their own pressing problems. The second practitioner article, by Lisa Bates, discusses her efforts to incorporate adult development theory into her GED math classes. Finally, Cynthia Zengler discusses the ways that the state of Ohio has tried to incorporate the Common Core State Standards (CCSS) and the College Readiness (CCR) Standards into its professional development efforts. She notes that while her article focuses on numeracy, their efforts provide a broader blueprint that others might follow.

The lead article in this issue's Forum is by Lynda Ginsburg. She lays out the dilemmas associated with the sometimes conflicting need to improve numeracy education for adults while also adhering to the myriad state and federal requirements. Donna Curry continues Ginsburg's discussion but focuses on students, instead of teachers. She notes that teachers are unable to provide the basic frameworks necessary for students to advance and still meet all of the many external requirements. The third article, by Melissa Braaten, continues this discussion, asking how teachers should decide what to teach, given all of the demands they face.

Finally, we are excited to include Nicole Taylor's review of an edited book by Susan Imel and Gretchen T. Bersch, *No Small Lives: Handbook of North American Early Women Adult Educators, 1925-1950*. This book is an historical examination of the roles that women have played in the development of the field of adult education. The last column in this issue, by David J. Rosen, focuses on numeracy and math websites. Dr. Rosen provides important information for instructors as they try to identify websites to aid them in the teaching of mathematics.

Sincerely,

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Evaluating Number Sense in Community College Developmental Math Students

Dorothea A. Steinke
NumberWorks

Abstract

Community college developmental math students ($N = 657$) from three math levels were asked to place five whole numbers on a line that had only endpoints 0 and 20 marked. How the students placed the numbers revealed the same three stages of behavior that Steffe and Cobb (1988) documented in determining young children's number sense. 23% of the students showed a lack of the concept of part-whole coexistence in this task. In two of three levels, lack of the concept was found to be significantly related to success (final grade of A, B, or C) in developmental math.

In her review of The Centre for Literacy's 2014 Summer Institute and its focus on data from PIAAC (Program for the International Assessment of Adult Competencies), Tighe (2014) commented that "research is needed ... to adequately design interventions to identify, target, and improve key component numeracy skills" (p. 66) among adult students. In the spirit of that comment, this article describes a practitioner-devised tool, and its use in a community college-sponsored research project to uncover which students appear to lack key numeracy components critical for understanding proportions, fractions and algebraic relationships. Stigler, Givvin, and Thompson (2009) reported a lack of conceptual understanding in those particular areas of pre-college-level math among community college developmental-level math students.

The purpose of this study was to identify how many students in developmental math classes may be lacking key developmental math concepts that standardized skills tests may fail to identify. These concepts are: 1) the "equal distance of 1" that exists between neighboring whole numbers, which is necessary for understanding abstract addition; and 2) part-whole coexistence (the parts and whole exist at the same time), which is necessary for understanding abstract subtraction, fractions and quantities in relationship (percents, ratios, functions, and more).

Over the years, through one-on-one interviews, the author had identified individuals who lacked one or both concepts 1) among High School Equivalency program (GED) math classes, 2) among a sample (N=11) of community college students, 3) among pre-service teachers, and 4) among prisoners transitioning back to society (Steinke, 1999; 2002; 2008). In all these populations, some individuals struggled to answer, or could not answer, the question $7 + ? = 25$ that was presented with physical objects and numerals, but not in written form.

Research Question

With the development of the much quicker Number Line Assessment tool, it became practical to attempt identification of concept-lacking adults with a much larger group. The research question was posed as: How many developmental math students lack one or both concepts at the start of the course, and what is the success rate of these students in developmental math classes?

The key purpose is identifying whole number concepts, rather than skills, that adults lack. To understand what these concepts are at the earliest level, we turn to research on young children's number sense carried out in the 1980s by Dr. Leslie Steffe and his colleagues at the University of Georgia.

Conceptual Framework

Mature number sense with whole numbers has been thought to appear around age 7 or 8 (Piaget, 1953). More recent research on brain development pushes that toward age 9 (Houdé et al., 2011), particularly for children who grow up in conditions of toxic stress (poverty and/or abuse) (Child Welfare Information Gateway, 2015). Other recent research relates children's math achievement to non-verbal number sense (Halberda, Mazocco, & Feigenson, 2008) and to placement of whole numbers on an empty number line (Booth & Siegler, 2008; Mundy

& Gilmore, 2009; Rouder & Geary, 2014; Schneider, Grabner & Paetsch, 2009).

In the 1980s, Steffe and his colleagues developed a model of primary-grade children's growth toward number sense (Steffe et al., 1983; Steffe, Thompson & Richards, 1982; Steffe, Richards & von Glasersfeld, 1978). Wright used a variation of Steffe's original model to assess larger groups of children (Wright, 1994). The outcome of those assessments was used to develop a math curriculum in Australia (Wright, 2003). *Math Recovery*, a "Response to Intervention" (RTI) program for the early grades in the United States, is a further extension of Steffe's early model (Miller, 2014; Wright, 2009).

Steffe with Cobb (Steffe & Cobb, 1988) later refined the original model to three stages: perceptual (concrete), figurative (representational) and abstract thinkers. This update was based on behaviors observed in one-on-one interviews in which children answered simple addition or missing addend questions. The concepts that allow students to progress from one stage of number sense to the next are: equal-sized units from one whole number to the next (the concrete-to-figurative transition); and part-whole coexistence (the figurative-to-abstract transition).

In their 3 Stages model, Steffe and Cobb defined children as Stage 1 (perceptual) when the children had acquired the number word sequence and could use it to count with one-to-one correspondence. The researchers documented these counting behaviors with Stage 1 primary grade students: 1) fingers are raised in a "block" for number patterns (i.e., all fingers go up at once); 2) objects must be seen in order to be counted (i.e., objects not in sight are not included in the count); and 3) counting to add starts from 1 each time (i.e., a "count all" strategy).

Older children and adults who understand each counting number as a separate item exhibit Stage 1 behaviors in one-on-one interviews (Steinke, 1999;

Steinke, 2001). These people understand numbers as labels of items in a certain order, like house numbers. For them, there is no exact quantitative distance from number to number. Number words belong to a category, like the names of fruit belong to a category.

Stage 2 also shows specific counting behaviors according to Steffe and Cobb (1988). The person: 1) raises fingers in sequence one after the other when counting; 2) can add unseen objects; 3) “counts on from” one of the addends when adding; 4) substitutes fingers, mental visualizations, or spoken words for unseen objects being counted; and 5) can add parts to find the whole without using physical objects. These behaviors, especially the ability to add unseen objects and “counting on from,” would indicate that Stage 2 children and adults have the sense that each counting number is the “same-sized 1 more” than the number before it. That is, since the increase from one number to the next is constant, it doesn’t matter where you begin counting when adding two groups of like items. Figure 1 contrasts the physical sense of number relationships between Stage 1 and Stage 2.

Stage 2 thinkers have the first major concept, “equal distance,” but lack the second, “part-whole coexistence.” Stage 3 thinkers have that second concept, namely, the understanding that a number exists as a whole and at the same time contains within it all the combinations of addends (the parts) that can be summed to create that whole. For example, 11 contains within it $4 + 7$ or $3 + 3 + 3 + 2$ and many other combinations while existing at the same time as the whole 11.

The important point here is the Stage 3 understanding that the parts and whole exist at the same time as opposed to the Stage 2 understanding that either the parts exist or the whole exists (Fig. 2). Steffe and Cobb (1988) also noted that Stage 3 children could give the solution to a missing addend (subtraction) question on the first try without using counters, and were confident that the answer was correct.

It is the grasp or lack of these two transition concepts (“equal distance” and “part-whole coexistence”) that the 5-digit number line assessment reveals. Other researchers have reported tasks with placement of a single number between two designated endpoints in order to show a relationship between students’ number sense and their physical placement of numbers relative to each other in space (De Hevia & Spelke, 2009; Longo & Lourenco, 2010). Using 5 digits uncovers much more, and in far less time than interviews.

Method

At a suburban community college, students taking developmental-level math courses (Basic math [whole numbers, fractions and decimals] [$N = 179$]; Pre-Algebra [$N = 167$]; Algebra 1 [$N = 311$]) were assessed for their sense of whole number relationships using an empty number line with endpoints zero and twenty. The college’s Institutional Review Board approved the study. Preliminary investigation with four developmental math classes of two different instructors had shown that not all students could place five given whole numbers on the empty line with reasonable accuracy.

The overall student population in the college is about 19% Hispanic and about 2% Black. In the classes that formed the assessment group, the amount of Hispanics was markedly above that 19%: 31.4% of students in Basic Math; 32.9% in Pre-algebra; and 23.3% in Algebra 1. Furthermore, the zip codes of 260 students in eleven Basic Math classes over a period of five years indicate that 33.5% lived in ZIP codes that are in the top 10% of Hispanic percentages of population nationally (U.S. Census Bureau Fact Finder); that 181 (69.6%) of the students in that ZIP code sample lived in two counties that have a higher poverty rate than the state figure (2013

poverty rates: State: 13.5%; County A: 18.4%; County B: 16.5%) (Ball, 2013); and that in 2013 the poverty rate for Hispanic households in the state was 2.5 times that for White non-Latinos (24.2% versus 9.0%) (Ball, 2013). The above information would seem to imply that the number of students who have grown up in and/or live in or near the poverty line is likely higher in developmental math classes than in the general population of this community college, given the higher percentage of Hispanics in those math classes. It is important to recognize this sub-group in the study population in light of recent reports of the adverse effects of living in poverty on the trajectory of children's brain development and learning. (Center on the Developing Child at Harvard University, 2016).

Student placement in developmental math was by standardized test (ACCUPLACER) or successful completion of a lower course (grade of C or higher). In the semester of the assessment, all on-campus sections of each course participated.

The test instrument was a line about 23 cm long, printed with the instructions on normal copy paper, with endpoints zero and twenty marked (Fig. 3). The decision to use a 0-to-20 line was based on earlier interviews with adults using Steffe and Cobb's model, where Stage of number sense could be determined with an oral missing addend question when the largest "whole" was 25 (Steinke, 1999). Also, using 20 allows those students able to do so to mentally picture the middle of the line as 10. The decision to use five numbers was based on an in-class experience with an adult student prior to developing the assessment. The given numbers were written in a vertical box and out of order – 17, 12, 2, 5, 1. The specific numbers were chosen based on: 1) avoiding 10 (a center benchmark) (Friso-van den Bos et al., 2015); 2) using only one other benchmark (either 5 or 15); 3) including 1 and 2 to show a person's sense of the "equal distance" concept (the distance from 0 to

1 and from 1 to 2 should be the same); 4) excluding numbers one more or one less than any benchmark beyond zero (thus excluding 4, 6, 9, 11, 14, 16, 19); and 5) using no consecutive numbers beyond 1 and 2 (thus excluding 3). From the remaining numbers (7, 8, 12, 13, 17, 18), two beyond 10 were chosen. This decision again was based on interviews; Stage 1 or weak Stage 2 adults began to struggle with missing addend questions in which the whole was greater than 10. The 12 and the 17 were chosen.

The assessment was usually done at the first class meeting of the semester and no later than the third class meeting. Participants were all the students present in class on the day of the assessment. After students received the assessment tool, the lead researcher or a result evaluator read the directions aloud while displaying the tool and physically pointing to the ends of the line (the zero and the twenty). If students had questions about how to proceed, a general remark such as "It's up to you." was given. Testing an entire class of up to 32 students took no more than ten minutes, including the time for distributing the assessment and reading the directions.

The tests were then analyzed separately for Stage of number sense by two different math instructors. The instructors met later to compare their separate results and arrive at a consensus on those assessments for which their original Stage placement differed. A template of the ideal (i.e., perfectly placed) location of each number was used to judge the accuracy of the responses.

Results

Stage 1 thinking appears on the assessments as positioning the five given numbers nearly equally across the number line (Fig. 4). This reflects the person's understanding that the numbers are in order but do not have a specific, physical size relationship.

It is also indicative of “must see them to count them” thinking. Numbers not listed appear to be ignored.

Stage 2 thinkers have an “either-or” understanding of the “part-whole” relationship (see Fig. 2). This causes them to focus on either the size of parts (the size of their personal, internal “1”) or the size of the entire line, but not the spatial relationship of both at the same time.

Stage 2 thinking appears on the assessments as numbers that are correctly proportionally spaced unto themselves, but that are not in the correct location on the entire line. Stage 2 thinking results in two main types of errors: 1) an obvious leftward skewing of the entire set of numerals, often to the left of the center of the line (Fig. 5a) or 2) a proportional spacing of the digits 1, 2, 5, and 12 too far to the left and a proportional spacing of 17 close to 20 (Fig. 5b). In both cases, the size of “1” is internal and individual for that person. Also, because Steffe and Cobb noted that Stage 3 thinkers in the interviews arrived at the correct answer on the first try and were certain of their answers, any corrections or erasures of the original placement of a number caused the assessment to be judged Stage 2 (Fig. 5c).

Contrast Stage 2 “either – or” thinkers’ assessments with those of Stage 3 thinkers who use the whole line as a reference and locate the numbers (the parts) within that distance (Fig. 6). People at Stage 3 may also mark the location of 10 and/or 15 on the line, a strong indication that they are thinking about the parts within and at the same time as the whole. Furthermore, Stage 3 thinkers have no erasures on their paper because, as Steffe and Cobb noted with Stage 3 children, they know their response is correct on the first try.

By far the majority of the assessments revealed a correct sense of number relationships on a number line. In many of these “correct” number lines, the 12 appears to be positioned slightly farther to the left than it should be. This is likely due to the well-

documented Spatial-Numerical Association of Response Codes (SNARC) effect. Researchers found that humans judge the distance between two larger neighboring numbers (like 12 and 13) to be less than the distance between two smaller neighboring numbers (like 2 and 3) (Dehaene, Bossini, & Giroux, 1993; Wood et al., 2008) even though both pairs of numbers are the same-sized “1” apart.

Analysis

Recapping the parameters used in evaluating a number line for Stage placement:

Stage 1 – The five given numbers are spaced fairly equally across the line.

Stage 2 – The five given numbers are spaced somewhat proportionally to each other, but not proportionally to the entire line on the first attempt. Specific Stage 2 indicators on an assessment are: 1) the numeral 12 placed left of the midline; 2) 1, 2, and 5 skewed toward zero and 12 and 17 skewed toward 20; 3) excessive space between 17 and 20.

Stage 3 – Reasonable spacing of the five given numbers on the first attempt, allowing for the SNARC effect; no erasures; and, in some results, marking the middle of the line as a reference point.

Inter-rater Reliability

When the instructors met to compare their individual analyses, there was strong initial agreement about which students were Stage 3. In the Algebra 1 assessments, one reviewer classified 215 results as Stage 3; the other agreed with 191 of those (89%). When reaching consensus on the remaining 24 assessments, only 3 were moved higher, from Stage 2 to Stage 3. There was also strong agreement about Stage 1: of the 6 in Algebra 1, four were agreed upon immediately, and two more by consensus.

Stage 2 was more complicated because of the SNARC effect and the variations of error types

(see Figure 5). How close to the exact location of the number did a student's placement have to be to qualify as Stage 3? Even so, in the Algebra 1 results, of the 59 assessments initially placed in Stage 2 by one reviewer, the second reviewer agreed with 55 of those placements, a 93% agreement rate. After discussion, a number of results were reclassified. If the two instructors could not agree on an example as Stage 2 or Stage 3, that assessment was labeled Stage 2.5. In reporting the results of this assessment set, all these uncertain-Stage results were put in the Stage 3 category. That means the final numbers reported here are very conservative.

Number Line Results

Tables 1, 2, and 3 show the percentages for the Stage of the students in each of the three courses. The first percentage is for all students who took the assessment (ALL). The second percentage includes only those students who received A, B, C, D, or F grade in the course (A to F grades) and excludes those who continued in the course after the census date but withdrew (W) prior to receiving a final grade. In fact, students who left before or after census had little to no affect on the overall percentages. Combining all three courses, 77% of those who took the assessment at the start of the term were Stage 3; 23% were not. At the end of the term, of those who had taken the assessment and received a letter grade, 78% were Stage 3 and 22% were not.

What is surprising is that there was a higher percentage of NOT Stage 3 students in Algebra 1 than in the lower-level courses. Looking at each course, the percentage of students NOT Stage 3 was 18% in Basic Math and 18% in Pre-algebra, while in Algebra 1 it was 28%. This implies that some students may be scoring high on the math placement exam even though they lack the background concept of "part-whole coexistence."

Stage of Number Sense and Math Course Success

Further analysis revealed that there is a difference in success rate in these math courses between those who have the part-whole concept (Stage 3) and those who do not. Success is defined as a final grade of A, B, or C. Including only those students who received grades of A through F, by a two-proportion z -test, the difference in success rate is significant in Pre-algebra at $p < .1$ ($p = .085$) and in Algebra 1 at $p < .05$ ($p = .039$). The difference was not statistically significant in Basic Math.

Furthermore, letter grades in all three courses for students who passed are skewed toward A and B for Stage 3 students and toward C for Stage 1 and 2 students as shown in Figures 7, 8, and 9. Note also that the percent of students who withdrew from each course after the census date but without receiving a grade (W) was higher for Stage 1 or 2 students than for Stage 3.

It is true that students may withdraw for job-related, family-related, or health-related reasons throughout the semester. However, anecdotal evidence, including from instructor gradebooks, indicated that students who withdraw just before the deadline (the end of week 13 of a 15-week semester) are more likely not to be passing the course at that point. Withdrawing avoids a poor grade. (Note that students who withdrew (W) are included in Figures 7, 8 and 9, making that the total number of students different from that in the Tables.)

"Rules" for Analysis of Future Tests

After the original "eyeball" analysis of the test results, a more rigorous analysis of the physical data was undertaken. Each marked point on each number line was measured by hand to the nearest .5 millimeter. When erasures were detected on the page, the original point(s) were measured as the person's response. The difference of each point

was computed plus-or-minus from the exact ideal location of that point on the number line. The ratio of the distances between each two neighboring points was also computed. The math instructors' visual classification of results was then compared with these numbers to attempt to find some general rules for reducing subjectivity in future number line assessment classification.

Stage 1 students' results generally were found to have ratios of the distance between neighboring numbers that approached 1 in at least three of the four comparisons where the ratio should not have been 1. This is the "equal spacing" that was noted in the visual classifying.

To attempt to find a rule for Stage 3, the Stage 3 assessments for Basic Math (137) and Pre-Algebra (146) were used. The mean of each of the five points from those results was taken as a benchmark and simple Standard Deviations (SD) from those benchmarks were computed. These parameters were then applied to the 311 Algebra 1 assessments.

It appeared that a criterion of all five points of the assessment falling within 1.5 SD from the benchmarks (that first set of Stage 3 means) might be a good sorting mechanism for Stage 3. In the Algebra 1 data, 184 of the 224 assessments identified by visual inspection and consensus as Stage 3 (165) or Stage 2.5 (19) (those uncertain results that were bumped to the higher level) meet the 1.5 SD criterion. That is, this 1.5 SD criterion sort matches 82% of the visual inspection sort.

These numerical results seem to support the trained math instructors' visual classification as being adequate as a quick first look for students at Stage 1 and Stage 3.

Stage 2 had no general numerical rule that could be deduced from the Stage 3 Standard Deviation data. This may be in part because of the variety of errors on Stage 2 number lines. Also, only 81 assessments from Algebra 1 were classified as Stage 2 by visual

inspection. That did not provide enough examples of each type of error to arrive at measurement-based rules for Stage 2 beyond "12 placed left of center." In the Algebra 1 course, 23 of the 81 Stage 2 results (28%) met this criterion.

A much larger set of assessments would need to be gathered to determine whether these criteria apply to the general population. Using newer technology (such as a pen that writes on a tablet or computer surface) and the GeoGebra software program (which can measure the distance between two points on a line automatically) a large-scale test would seem to be feasible.

Significance

The concept of part-whole coexistence is critical for understanding proportions, fractions and algebraic relationships. The concept is also central to the College and Career Readiness Standards (CCRS) (Pimental, 2013) around which the new adult high school equivalency tests are built. Students' positioning of non-sequential whole numbers on the empty line appears to reveal whether they grasp that concept and have arrived at mature number sense.

The results of this study suggest that over 20% of developmental math students in this sample have not. This is in line with results from the 2003 National Assessment of Adult Literacy (NAAL) (which included numeracy) showing 22% of adults in the United States at below-basic level in math (U.S. Department of Education, 2011). The recent PIAAC international test of adult numeracy (U.S. Department of Education: PIAAC, 2014) indicated similar math deficiencies: 30% of American adults were below or at Level 1, compared to the international average of 19%.

Remediation

How can this picture be changed? Adult students are apt to resist revisiting primary-grade-level

concepts (see Figs. 1 & 2) if instruction is undertaken in a purely mathematical context.

Effecting conceptual change is more likely to be successful when new ideas are linked to students' personal experiences. Below are brief descriptions of some of the ways this instructor has addressed key concepts, including the meaning of the equals sign. Changing students' understanding of that symbol from "operation" to "relationship" (Wheeler, 2010; Knuth et al., 2008) is required prior to addressing the equivalency relationship implicit in the part-whole coexistence and "equal distance" concepts.

1) Equals sign: Use the full name and nickname of several students. On the board, write an equals sign between each set of names, stressing "different name, same person." Follow up with examples of equivalent expressions with different operations, such as $17 - 9 = 4 \times 2$ and "different name, same amount."

2) Part-whole coexistence: Have students name the parts of an object (a chair, a car). Ask if the object is complete if a part is missing. Ask if the parts continue to exist within the object when speaking of the whole object. Follow up with missing addend and missing factor word problems with misleading "key words." Encourage students to think of the number information in the problems in terms of the part-whole coexistence relationship.

3) Equal distance between whole numbers: Ask students to trace with a finger the spaces between the marks of a 1-unit number line *at a steady beat* (Fig. 10). Use a digital metronome (marking equal spaces of time) set at the students' comfortable body speed. Be sure students place their tracing finger on the zero mark to start. Follow up with lessons on line graphs or the coordinate grid, emphasizing the equal spaces between the lines, not the digits.

Suggestions for Further Research

The revelation of the degree to which the two concepts, equal distance and part-whole coexistence,

are lacking in adult students makes this area ripe for further investigation. The utility and reliability of this number line assessment could be compared to that of standard computation-based math placement exams when determining a student's appropriate starting point for math remediation and/or course placement. Another interesting avenue would be to compare number line assessment results with tests of critical thinking skills or reading comprehension, both of which also require considering the parts and the whole at the same time.

The topic of remediation for students lacking the concepts is also open for research. What tools and materials are most effective? Will whole-class instruction work? Does remediation with adults need to be one-on-one?

Implications for the Field

Current mainstream adult basic education math texts and college developmental math texts do not explicitly teach either of the missing concepts, "equal distance" and "part-whole coexistence" with whole numbers, and that concept's necessary precursor, the equals sign as relationship. It would seem the texts assume that adults grasp these concepts. Such an assumption may exist in math curricula as early as fourth grade, about age 9. That is the age at which the brains of students living in the toxic stress of poverty (Child Welfare Information Gateway, 2015) are perhaps just beginning to grow the connections that allow the student to keep two things in mind at the same time, a pre-requisite for understanding part-whole coexistence. This brain growth often happens for children living in more secure environments at about age 8 (Rueda et al., 2004), which is 3rd grade, and seems to be secure for 9-year-olds (Poirel et al., 2012), which is 4th grade.

As noted earlier, many of these developmental math students likely come from low socio-economic backgrounds, where toxic stress delays "normal"

brain development. Other students may have been the youngest in their class (or nearly so), so their brain development was later than their classmates, the “relative age effect” documented by Bedard and Duhey (2006). Whatever the cause, the Stage 1 and Stage 2 adult students were not able to grasp the concepts when they were presented in the primary grades. Until the brain development is there, teaching these two concepts is like expecting a color-blind person to be able to learn to distinguish between lime green and chartreuse.

The ultimate solution would seem to lie in aligning the elementary math curriculum with students’ neurological development rather than chronological age. The system needs to wait until the brain is ready before presenting abstract concepts that require part-whole thinking. In the meantime, the quick assessment presented here may be a useful tool for teachers to determine the true root of many adults’ difficulty with part-whole relationships in fractions and decimals, and to lead to appropriate explicit instruction in those concepts for those adults. Such instruction will meet the needs of more students and allow them to be more successful in math. ❖

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References

- Ball, A. (2013). *State of Working Colorado*. Denver, CO: Colorado Center on Law and Poverty.
- Bedard, K., & Duhey, E. (2006). The persistence of early childhood maturity: International evidence of long-run age effects. *Quarterly Journal of Economics*, 121 (4), 1437-1472.
- Booth, J. & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development*, 79(4), 1016-1031.
- Center on the Developing Child at Harvard University (2016). 8 Things to Remember about Child Development. Retrieved from <http://www.developingchild.harvard.edu>
- Child Welfare Information Gateway (2015). *Understanding the effects of maltreatment on brain development*. Washington, DC: U.S. Department of Health and Human Services – Children’s Bureau. Retrieved from <https://www.childwelfare.gov/pubs/issue-briefs/brain-development/>
- De Haene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General*, 122(3), 371-396.
- Friso-van den Bos, I., Kroesbergen, E. H., Van Luit, J. E. H., Xenidou-Dervou, I., Jonkman, L. M., Van der Schoot, M., & Van Lieshout, E. C. D. M. (2015). Longitudinal development of number line estimation and mathematics performance in primary school children. *Journal of Experimental Psychology* 134, 12-29.
- De Hevia, M-D. & Spelke, E. S. (2009). Spontaneous mapping of number and space in adults and young children. *Cognition*, 110(2), 198-207.
- Halberda, J., Mazocco, M., & Feigenson, O. L. (2008). Individual differences in non-verbal number acuity correlate with math achievement. *Nature*, 455, 655-668.
- Houdé, O., Pineau, A., Leroux, G., Poirel, N., Perchey, G., et al. (2011). Functional magnetic resonance imaging study of Piaget’s conservation-of-number task in preschool and school-age children: A neo-Piagetian approach. *Journal of Experimental Child Psychology*, 110(3), 332-46. doi: 10.1016/j.jecp.2011.04.008
- Knuth, E. J., Alibali, M. W., Hattikudur, S., McNeil, N. M., & Stephens, A.C. (2008). The importance of equal sign understanding in the middle grades. *Mathematics Teaching in the Middle School*, 13(9):514-519.
- Longo, M. R. & Lourenco, S. F. (2010). Bisecting the mental number line in near and far space. *Brain and Cognition*, 72(3), 362-367. doi: 10.1016/j.bandc.2009.10.016.
- Miller, C. (2014). Math Recovery Overview: An elementary school implementation of an early intervention program to identify and service of low-attaining mathematics students. White Paper prepared by U.S. Math Recovery Council. Apple Valley, MN. Retrieved from <http://www.mathrecovery.org/pdfs/how-it-works/Math-Recovery-Research-White-Paper.pdf>
- Mundy, E. & Gilmore, C.K. (2009). Children’s mapping between symbolic and nonsymbolic representations of number. *Journal of Experimental Child Psychology*, 103(4), 490-502.
- Piaget, J. (1953). How children form mathematical concepts. *Scientific American*, 189(5), 74-79.
- Pimental, S. (2013). *College and career readiness standards for adult education*. Prepared for the U.S. Department of Education, Office of Vocational and Adult Education. Washington, DC: MPR Associates, Inc. Retrieved from <http://www.sbctc.ctc.edu/college/abe/CCRStandardsAdultEd.pdf>
- Poirel, N., Borst, G., Simon, G., Rossi, S., Cassotti, M., Pineau, A., & Hourdé, O. (2012). Number conservation is related to children’s prefrontal inhibitory control: An fMRI study of a Piagetian task. *PLoS ONE*, 7(7): e40802. doi: 10.1371/journal.pone.0030802
- Rueda, M. R., Fan, J., McCandliss, B. D., Halparin, J. D., Gruber, D. B., Lercari, L. P., & Posner, M. (2004). Development of attentional networks in childhood. *Neuropsychologia*, 42(8), 1029-1040. Retrieved from <http://www.sacklerinstitute.org/cornell/people/jin.fan/publications/>
- Rouder, J. N. & Geary, D. C. (2014). Children’s cognitive representation of the mathematical number line. *Developmental Science*, 17(4), 525-536. doi: 10.1111/desc.12166
- Schneider, M., Grabner, R., & Paetsch, J. (2009). Mental number line, number line estimation, and mathematical achievement: Their interrelations in grades 5 and 6. *Journal of Educational Psychology*, 101(2), 359-372.
- Steffe, L., & Cobb, P., with von Glasersfeld, E. (1988). *Construction of arithmetical meanings and strategies*. New York, NY: Springer-Verlag
- Steffe, L., Richards, J., & von Glasersfeld, E. (1978). Experimental models for the child’s acquisition of counting and of addition and subtractions. In K.C. Fuson & W.E. Geeslin (Eds.), *Explorations in the modeling of the learning of mathematics* (27-44). Columbus, OH: ERIC/SMEAC Center for Science, Mathematics, and Environmental Education.
- Steffe, L., Thompson, P., & Richards, J. (1982). Children’s counting in arithmetical problem solving. In T. Carpenter, J. Moser, & T. Romberg (Eds.), *Addition and subtraction: A cognitive perspective* (211-223). Hillsdale, NJ: Erlbaum.
- Steffe, L., von Glasersfeld, E., Richards, J., & Cobb, P. (1983). *Children’s counting types: Philosophy, theory, and application*. New York, NY: Praeger Scientific.
- Steinke, D. (1999). Adults’ understanding of number: Three groups interviewed for Steffe and Cobb’s 3 stages. *MPAEA Journal of Adult Education*, 27(1), 24-29.

- Steinke, D. (2001). Part-whole concept understanding in a population of 2nd and 3rd graders. In R. Speiser, C. A. Maher & C. N. Walter (Eds.). *Proceedings of the twenty-third annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (Vol.1), 331-319. Columbus, OH: Eric Clearinghouse for Science, Mathematics, and Environmental Education. ERIC accession number: ED482957
- Steinke, D. (2002). Preservice teachers' part-whole concept understanding: Two examples. In D. S. Mewborn, P. Sztajn, D. Y. White, H.G. Wiegel, R. L. Bryant & K. Nooney (Eds.). *Proceedings of the twenty-fourth annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (Vol. 3), 1226-1229. Columbus, OH: Eric Clearinghouse for Science, Mathematics, and Environmental Education. ERIC accession number: SE 066 889
- Steinke, D. (2008). Using part-whole thinking in math. *Focus on Basics*, 9(A), 1,3-8. Retrieved from http://www.ncsall.net/fileadmin/resources/fob/2008/fob_9a.pdf
- Steinke, D. (2015). Evaluating number sense in workforce students. *MPAEA Journal of Adult Education*, 44(1), 1-8.
- Stigler, J. W., Givvin, K. B., & Thompson, B. J. (2009). What community college developmental mathematics students understand about mathematics. THE CARNEGIE FOUNDATION FOR THE ADVANCEMENT OF TEACHING: Problem Solution Exploration Papers. http://cdn.carnegiefoundation.org/wp-content/uploads/2013/05/stigler_dev-math.pdf (A version of the paper was published in the *MathAMATYC Educator*, v1 n3 p4-16 May 2010.)
- Tighe, E. L. (2014). Broadening our perspectives on adult literacy, numeracy, and problem-solving skills with PIAAC Data: A commentary on The Centre for Literacy's 2014 Summer Institute. *Journal of Research and Practice for Adult Literacy, Secondary, and Basic Education*, 3(3), 63-69.
- U.S. Department of Commerce, Census Bureau, Fact Finder. ZIP Code information retrieved from http://factfinder.census.gov/faces/nav/jsf/pages/community_facts.xhtml#
- U.S. Department of Education, National Center for Education Statistics. *PIAAC: What the Data Says about the Skills of U.S. Adults*. Washington, D.C., 2014. Retrieved from: http://static1.squarespace.com/static/51bb74b8e4b0139570dddf020/t/535fdfdb4b032825e56cbf8/1398792155103/PIAAC+Results+Summary+Brochure_Final_041814.pdf
- U.S. Department of Education, Office of Vocational and Adult Education. *Building on Foundations for Success: Guidelines for Improving Adult Mathematics Instruction*. Washington, D.C., 2011. Retrieved from <http://lincs.ed.gov/publications/pdf/AdultNumeracyReportFinal2011.pdf> Citing: U.S. Department of Education, National Center for Education Statistics. The National Assessment of Adult Literacy, Washington, D.C., 2003. http://nces.ed.gov/naal/kf_demographics.asp
- Wheeler, G. D. (2010) Assessment of college students' understanding of the equals relation: Development and validation of an instrument. All Graduate Theses and Dissertations. Paper 815. Retrieved from: <http://digitalcommons.usu.edu/cgi/viewcontent.cgi?article=1811&context=etd>
- Wood, G., Willmes, K., Nuerk, H-C., & Fischer, M. H. (2008). On the cognitive link between space and number: A meta-analysis of the SNARC effect. *Psychology Science*, 50(4), 489-525.
- Wright, R. J. (1994). A Study of the numerical development of 5-year-olds and 6-year-olds. *Educational Studies in Mathematics*, 26, 25-44.
- Wright, R. J. (2003). Mathematics Recovery: A program of intervention in early number. *Australian Journal of Learning Disabilities*, 8(4), 6-11.
- Wright, R. J. (2009). An overview of Mathematics Recovery. In A. Dowker (Ed.), *Mathematics difficulties: Psychology, neuroscience and intervention*. San Diego, CA: Elsevier.

Table 1— Percent of BASIC MATH students at Stage 3, Stage 2, and Stage 1

Based on the Number Line Assessment of Number Sense				
Basic Math	ALL		A to F grades	
NUMBER	179		160	
Stage 3	146	81.6%	131	81.9%
Stage 2	17	9.5%	15	9.4%
Stage 1	16	8.9%	14	8.8%

Table 2— Percent of PRE-ALGEBRA students at Stage 3, Stage 2, and Stage 1

Based on the Number Line Assessment of Number Sense				
Pre-Alg.	ALL		A to F grades	
Number	167		137	
Stage 3	137	82.0%	114	83.2%
Stage 2	18	10.8%	14	10.2%
Stage 1	12	7.2%	9	6.6%

Table 3—Percent of ALGEBRA 1 students at Stage 3, Stage 2, and Stage 1

Based on the Number Line Assessment of Number Sense				
Algebra 1	ALL		A to F grades	
NUMBER	311		247	
Stage 3	224	72.0%	182	73.7%
Stage 2	81	26.1%	61	24.7%
Stage 1	6	1.9%	4	1.6%

For Tables 1, 2, and 3:

ALL includes students who dropped before census or withdrew with no grade after census.

A to F includes only those tested who also received a letter grade.

Figure 1—Stage 1 versus Stage 2 understanding of number relationships

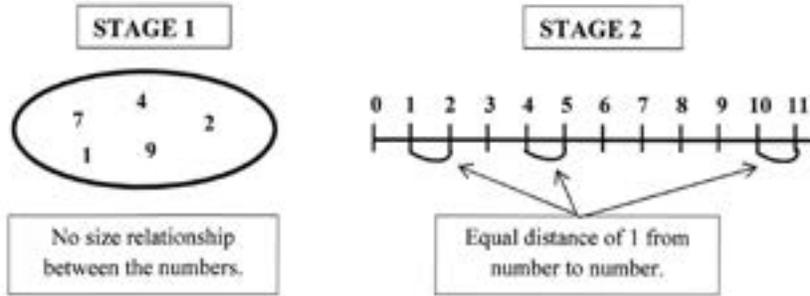


Figure 2—Stage 2 versus Stage 3 understanding of number relationships

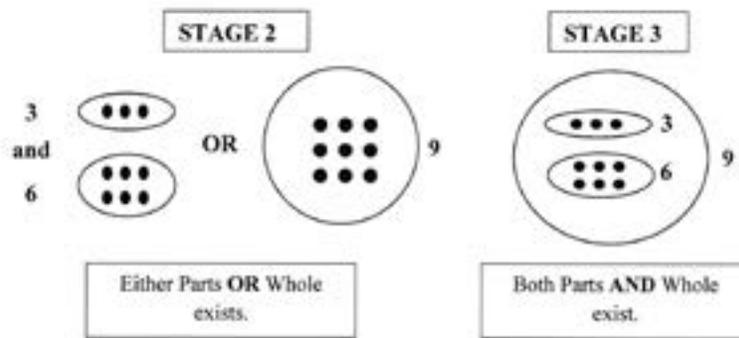


Figure 3—Assessment Tool

Math - Preliminary Assessment

NAME _____

The line below starts at zero and ends at twenty.
 All the numbers in the boxes at the right belong somewhere on the line.
 Make a mark on the line where you think a number goes.
 Write the number by the mark you make for it.

17
12
2
5
1

0 _____ 20

Figure 4—Stage 1 Number Line

All numbers nearly equally spaced across the line.

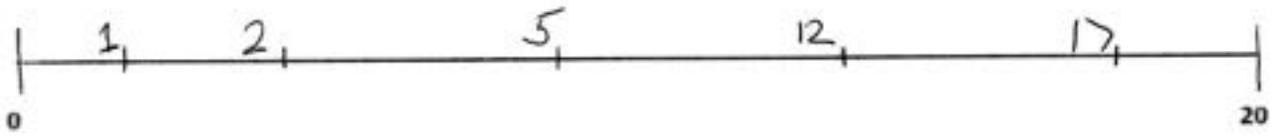
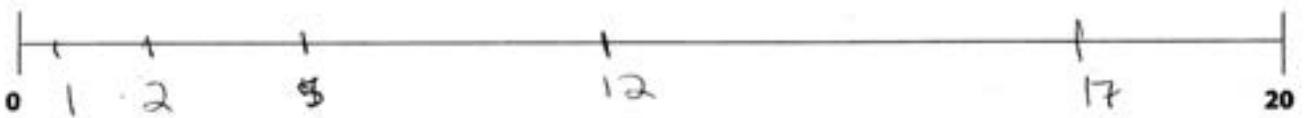


Figure 5—Stage 2 Number lines

a) Numbers skewed left



b) Numbers skewed toward ends; 12 left of center



c) Excess space between 5 and 12; correction of placement



Figure 6—Stage 3 Number Line

Correct relationship of parts within the whole line.



Figure 7—Basic Math Grade Distributions

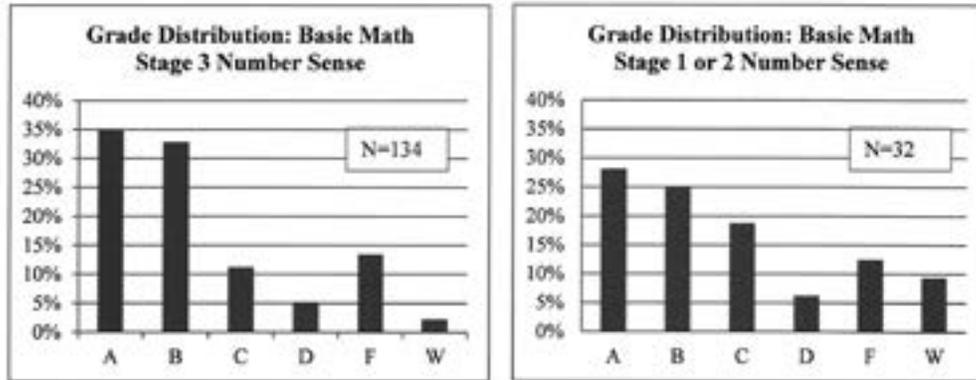


Figure 8—Pre-algebra Grade Distributions

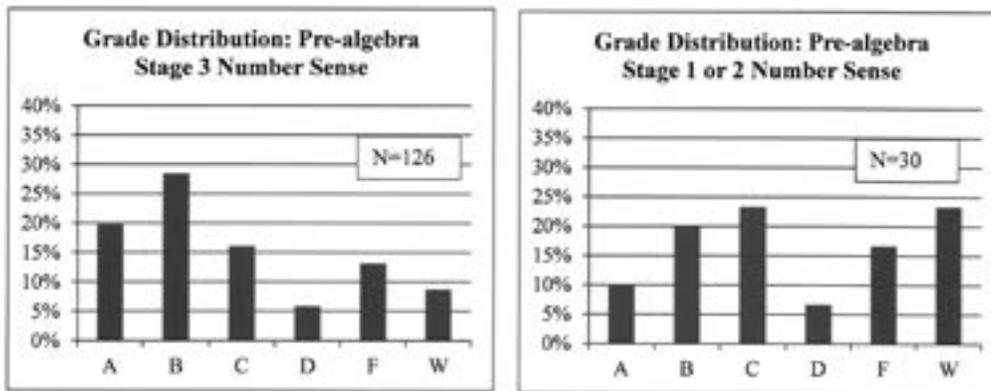


Figure 9—Algebra 1 Grade Distributions

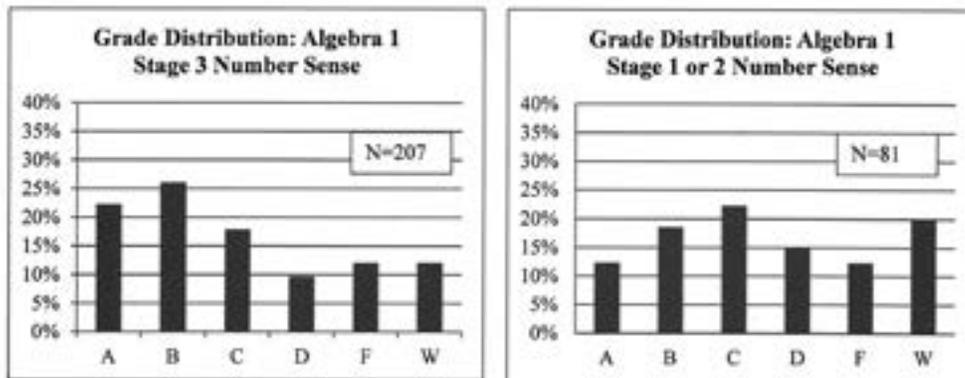
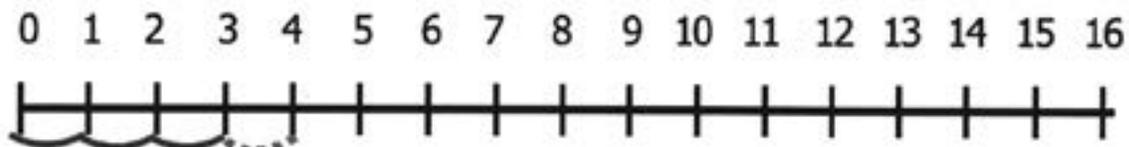


Figure 10—Sample of number line with guide to trace spaces



The Case for Measuring Adults' Numeracy Practices

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Abstract

In this article, we make the case for the development of a numeracy practices measure in the light of a review of relevant research and extant measures. We argue that a numeracy practices measure would acknowledge and validate adult learners' practice gains and inform teaching geared to their circumstances, needs and interests.

In New Zealand, there is a robust infrastructure supporting adult literacy and numeracy education and training. Professional development is built around the "three knowings": know the learner; know the demands; know what to do (National Centre of Literacy & Numeracy for Adults, 2011). Learners' progress is measured by an online adaptive proficiency measure, the Literacy and Numeracy for Adults Assessment Tool (TEC, 2016). Adult numeracy learners often mention to their tutors that since joining a program they work out the cost of shopping, help their children with their mathematics homework, or perform work calculations and estimations that they previously avoided. However, these "practice" gains may not be reflected in improved scores on proficiency assessments, to the frustration of tutors and learners alike. In response, we undertook a project scoping the development of a measure of adults' numeracy and literacy practices for the New Zealand

Ministry of Education. Our challenge is to find a way of measuring such practices in a robust, evidence-based, culturally-sensitive, ethical, practicable, and cost-effective way, in order to inform teaching and recognize learning.

Here we outline selected aspects of our work. We present a review of relevant literature and set out the case for a measure of adults' numeracy and literacy practices before briefly reviewing a selection of existing measures which encompass elements of numeracy and literacy practice measurement, and recommending ways forward.

Measuring Numeracy and Literacy Practices

The idea of measuring numeracy and literacy practices is gaining traction in various places around the world. For example, in the United States, Reder (2013) argues that measuring engagement with numeracy and literacy practices would be a good way of tracking change during and after engagement with learning programs, complementing proficiency measures. Similarly, Esposito, Kebede, and Maddox (2012, p. i), in Mozambique, contend that “measuring preferences and weighting of literacy practices provides an empirical and democratic basis for decisions in literacy assessment and curriculum development, and could inform rapid educational adaptation to changes in the literacy environment.”

Our focus in this article is primarily on numeracy, and we are mindful of the fact that terminology around numeracy is complex (Coben et al., 2003). Numeracy is often treated as an aspect of literacy in research and policy literature, with scant regard to its particularities. We contend that numeracy should be taken seriously on its own terms, with an equal, rather than a subservient relationship to literacy (Coben, 2006, p. 103). Accordingly, where it is necessary to consider both numeracy and literacy in this paper

we have chosen to reverse the normal order (i.e., “literacy and numeracy”) to emphasize this point. This is in keeping with numeracy's emergence onto the international stage in recent years. For example, “quantitative literacy” was specified as one of “three domains of literacy skills” in the Organization for Economic Cooperation and Development's (OECD's) International Adult Literacy Survey (IALS) in the 1990s (OECD & Statistics Canada, 2000, p. x) but more recent international surveys of adult skills have specified “numeracy” as an information processing skill in its own right. The definition of numeracy in the latest such survey, the Survey of Adult Skills in the Program for the International Assessment of Adult Competencies (PIAAC) is one we find helpful because of its orientation towards practice:

Numeracy is the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life. (PIAAC Numeracy Expert Group, 2009, p. 55)

The focus on use and engagement in the PIAAC definition of numeracy is somewhat at odds with the focus in much of the policy literature on adult numeracy and literacy as technical skills producing human capital outcomes (Keeley, 2007; Sen, 1997). Street (1984, p. 29) terms this the “autonomous model,” which he characterizes as “supposedly technical and neutral.” By contrast, the academic literature on adult numeracy and literacy is weighted towards a social practice perspective (Street, 1984; Tett, Hamilton, & Hillier, 2006). This perspective aligns with what Street calls the “ideological model,” in which literacy is seen as culturally-sensitive, context-dependent and embedded in power relations. Proponents of this approach tend to value social capital (Bourdieu, 1976) as an intended outcome of public policy. Debate is

polarized at best; at worst, it is absent. We cross this divide. We see numeracy and literacy as both social practices and technical skills, productive of both social and human capital. We agree with Schuller (2001) that these forms of capital have complementary roles in lifelong learning. He contends that the use of social capital opens up possibilities for the exploration of contemporary paradoxes, such as: the dominance of individual choice; policy consensus on the importance of lifelong learning; demands for accountability and evaluation in the public sphere; and technically more sophisticated measurement methodologies. The last of these is particularly relevant to our project scoping the development of a measure of adults' numeracy and literacy practices. We are interested in what adults do with their numeracy and literacy in a range of contexts, thus, our approach fits within a social practices perspective.

The emergence of a social practices perspective on numeracy and literacy is an example of the "practice turn" in contemporary social theory (Knorr Cetina, Schatzki, & von Savigny, 2005). Writing in this mode, Schatzki (2012, pp. 14-15) describes practice as "an open-ended, spatially-temporally dispersed nexus of doings and sayings" that takes place in a teleological hierarchy for which the "practicer" has an end in view. He contends that "A practice embraces all the activities contained in such teleological hierarchies: the activities and states of existence for the sake of which people act, the projects, i.e., actions they carry out for their ends, and the basic doings and sayings through which they implement these projects." Furthermore, "a practice's activities are organised by practical rules, understandings, teleoaffective structures, and general understandings." We consider practice in this light.

Practice necessarily takes place in a particular situation so we want to measure 'situated practice' (Balatti, Black, & Falk, 2006; Hutchings, Yates, Isaacs, Whatman, & Bright, 2012; Reder, 2008). Practice is

also goal-directed, since adults are likely to have a reason for improving their skills (Stewart, 2011; Waite, Evans, & Kersh, 2014). These goals may be extrinsic, such as to improve skills for work, at home or in the community, or intrinsic: for self-improvement. For example, adult numeracy learners in England stated that they attended classes: "to prove that they have the ability to succeed in a subject which they see as being a signifier of intelligence; to help their children; and for understanding, engagement and enjoyment;" goals such as gaining a qualification or coping better with mathematics in everyday life were a minor incentive (Swain, Baker, Holder, Newmarch, & Coben, 2005, p. 9). Following Schatzki (2012), we characterize numeracy and literacy practice as an open-ended, situated, spatially-temporally dispersed nexus of goal-directed doings and sayings involving numeracy and literacy.

Social practice theories of adult numeracy and literacy take a number of forms (Perry, 2012) and draw on a range of disciplines with a correspondingly wide variety of methodologies. For example, the "new literacy studies" (NLS) developed by Street and others (Hull & Schultz, 2001) draw mainly on sociology, socio-linguistics and anthropology and favor ethnographic approaches. As the name suggests, NLS is stronger on literacy than numeracy, as Street's (2003) review attests. Lave and Wenger's (1991) theories of situated cognition and communities of practice draw on social anthropology and psychology, while cultural historical activity theory (CHAT) (Engeström, 2001) draws on the work of the psychologists Leont'ev (1969/1995) and Vygotsky (1962, 1978). Reder's (1994) practice-engagement theory also draws on Vygotsky. Reder contends that literacy skills and reading practices develop best within specific practice contexts. Practice-engagement theory specifies the relationships between "expressed literacy choices/preferences and perceived social meanings" in a

detailed, practice-specific way, emphasizing “the patterns of individuals’ access to and participation in various roles within as well as across cultural groups” (Reder, 1994, p. 59). It acknowledges the possibility of continued development or decline of numeracy and literacy skills in relation to the affordances of any given situation and the individual’s use of numeracy and literacy.

Maddox and Esposito (2011, p. 1319) propose a “capabilities approach,” in which “literacy can be understood not simply as cognitive abilities or competencies, but as a set of ‘functionings’ (as beings and doings), or the potential to function.” They note that the concept of “literacy functionings” is similar to that of “literacy practices” in the ethnographic literature (citing Street, 1993), drawing attention to the social uses of literacy, and the production and embodiment of social identities.

These perspectives have generated corresponding methodologies and units of analysis. For example: for Vygotsky the unit of analysis is individual activity; for CHAT researchers it is the activity system (Engeström, 2001); for researchers working in a situated cognition perspective it is “practice,” “community of practice,” and “participation.” Street distinguishes between “literacy events” and “literacy practices” as units of analysis, such that literacy practices are the “broader cultural conception of particular ways of thinking about and doing reading and writing in cultural contexts” (Street, 2000, p. 11), whereas “literacy events” are discrete situations in which people engage with reading or writing (Heath, 1982). Similarly, Barton and Hamilton (1998) describe “literacy events” as activities in which literacy has a role. Purcell-Gates and colleagues (2000, p. 3) define literacy events as “the reading and writing of specific texts for socially-situated purposes and intents.” In this perspective, while literacy practices are unobservable, the associated literacy events are observable. This

distinction is problematic for numeracy since it may be invisible to those engaged in it (Coben, 2000; Keogh, Maguire, & O’Donoghue, 2012; Noss & Hoyles, 1996) and ‘numeracy events’ might or might not be observable, depending, for example, on whether someone uses a calculator, counts on their fingers or calculates mentally, or paces out a space rather than judging distance by eye.

As Reder (2016) notes, while social practices proponents have offered strong critiques of interpretive and policy frameworks reliant on standardised test scores alone, large scale practical alternatives have not been proposed. He argues that this is particularly problematic for the development of more effective adult numeracy and literacy programs which would benefit from richer measures of learner progress and program evaluations based on those measures. We are seeking to develop such a richer, technically more sophisticated measurement methodology, in Reder’s (2016) and Schuller’s (2001) terms. In the next section we set out the case for such a measure.

The Case for a Measure of Adults’ Numeracy and Literacy Practices

Our rationale for the development of a measure of adults’ numeracy and literacy practices is evidence-based, as follows.

1. The development of literacy and numeracy proficiency over time is strongly associated with adults’ engagement in literacy and numeracy practices.

There is evidence from the U.S. Longitudinal Study of Adult Learning (LSAL) and elsewhere that the development of adults’ numeracy and literacy proficiency over time is strongly associated with their engagement in numeracy and literacy practices, bearing out the prediction of practice engagement theory that engagement in numeracy and literacy

practice leads to growth in proficiency (Reder, 1994; Sheehan-Holt & Smith, 2000). LSAL found that “Adults at similar proficiency levels at one point in time wind up many years later at different proficiency levels depending in part on their earlier levels of engagement in literacy practices” (Reder, 2009, p. 47).

2. Educational programs that increase learners’ engagement in numeracy and literacy practices show improved outcomes for learners in terms of increased numeracy and literacy proficiency and future life benefits.

Of particular interest here is the direction of causality demonstrated by LSAL, where “The sequence of observed changes makes it clear that program participation influences practices rather than vice-versa” (Reder, 2008, pp. 3-4).

Similarly, research in New Zealand found that learners reported changes in their work practices stemming from their participation in a workplace program, including, for example:

“I don’t have to use my fingers. I can work out how many there are on a pallet [when multiplying rows of products]”

“I’m now working out the volume of concrete. The engineers used to come out, now they just double-check it.”

(Department of Labour, 2010, pp. 56-57)

In Canada’s UPskill initiative, Gyarmati et al. (2014) found that when workers developed their workplace numeracy and literacy skills they were able to transfer them into their wider family and community lives, showing improvements on behavioral and numeracy and literacy practice indicators.

PIAAC data also indicate a relationship between proficiency and practice in that:

adults who practice their literacy skills nearly every day tend to score higher (*sic*), regardless of their level of education. This suggests that there might be practice effects independent of education effects that influence proficiency.

(OECD, 2013, p. 212)

For Sticht (2013), the PIAAC results confirm “the three-way interaction of education, literacy skill, and engagement in literacy practices” which he terms the “triple helix’ of literacy development.” He explains this term as follows: “By this we meant that education produces some literacy skill, that leads to more practice in reading, which helps in the pursuit of more education, leading to more skill, leading to more engagement in reading, and so forth.”

The extent to which numeracy and literacy practices build from participation in programs is contingent on a range of factors. For example, using authentic contexts in learning programs increases the likelihood that there will be improvements in practices (Purcell-Gates, Degener, Jacobson, & Soler, 2002; Reder, 2008). Vaughan (2008) adds that learning must be meaningful for it to be practiced in a valued way. Adults need to use their learning in different contexts, transferring learning from education into other contexts such as the workplace, a process which requires time and support (Eraut, 2004). For numeracy, Evans (1999) notes that transfer is not dependable but neither is it impossible. He recommends designing pedagogic approaches that will facilitate transfer, building bridges between practices within and outside education. With such factors in place, educational programs may ‘jump-start’ adults into engaging in numeracy and literacy practices that

use and subsequently further develop their numeracy and literacy skills.

Accordingly, a practices measure would support teaching and learning that is more attuned to the type of engagement that research shows is effective in building proficiency over the long term (Reder, 2012). Engagement in numeracy and literacy practices is crucial if the numeracy and literacy of those with low skills are to improve and adults with the lowest numeracy and literacy skills have less opportunity than those with higher skills to perform workplace tasks that involve numeracy or literacy on a regular basis (Dixon & Tuya, 2010). These proficiencies are directly relevant to adults' prospects, wellbeing and quality of life (Reder, 2016). LSAL (Reder, 2012), UK research (Bynner & Parsons, 2009), and large-scale international adult numeracy and literacy assessments, most recently PIAAC (OECD, 2016a) exhibit strong relationships among numeracy and literacy proficiency, employment and earnings and other positive life outcomes. Numeracy skills decline during periods of unemployment, perhaps because some numeracy skills are used only at work rather than being reinforced through practice in everyday life (Bynner & Parsons, 1998).

3. An effective measure is needed to capture learners' progress over the relatively short time periods typical of literacy and numeracy programmes

The LSAL project in the United States found no relationship between change in proficiency and program participation “over the relatively short time intervals typical of program participation and of program accountability and improvement cycles” (Reder, 2011, p. 4). Small reported differences may be recorded in pre- and post-program tests but such proficiency gains can also be made by non-participants (Reder, 2008). However, LSAL found

that adult numeracy and literacy programs do “have demonstrable impact on measures of literacy and numeracy practices” over relatively short time-periods (Reder, 2012, p. 5). Similarly, analysis of New Zealand's Assessment Tool data shows little correlation between time on-program and proficiency gain in the short term (Lane, 2013a, 2013b, 2014). A practice measure would fill this information gap.

4. A practices measure could encompass numeracy and literacy practices occurring as part of adults' engagement with digital technologies.

There is growing recognition of the importance of the ability to use technology to solve problems and accomplish complex tasks, what PIAAC terms “Problem-Solving in Technology-Rich Environments” (PS-TRE) (OECD, 2016b). Numeracy and literacy are integral to PS-TRE and digital skills more generally and engagement with ubiquitous digital technology is a feature of many adults' practices, for example, to access products and services online. Potential benefits of improving adults' digital skills include productivity gains and facilitating fuller participation in society by marginalised groups (Bunker, 2010) and learning with and through technology engages and retains learners (Davis et al., 2010; Thomas & Ward, 2010). A recent UK report highlights the need to increase the focus on “digital literacy” skills and for these to be seen as complementary to numeracy and literacy skills (House of Lords Select Committee on Digital Skills, 2015). A practices measure could encompass numeracy and literacy practices naturally, as part of adults' engagement with digital technologies.

5. A literacy and numeracy practices measure is intrinsically sensitive to learner diversity

Because a practices measure focuses on what

adults do, it necessarily encompasses diverse learners and the diverse contexts in which numeracy and literacy are practiced. It should therefore be sensitive to cultural and linguistic diversity and differentiated power relations (Perry, 2012). It should also be sensitive to learning difference, since conditions such as dyslexia and dyscalculia may directly affect adults' engagement in numeracy and literacy practices (DfES, 2006).

In summary, we argue that a measure of numeracy and literacy practices would give a fuller picture of the capabilities of diverse adult learners, complementing proficiency data and attuned to the exigencies of learning programs. Once practices are measured their importance is likely to be recognized by tutors and an increased focus on practices in learning programs is likely to lead to improved outcomes for learners in terms of increased numeracy and literacy proficiency and future life benefits.

Is a Measure of Adults' Numeracy and Literacy Practices Already Available for Use with Adult Learners?

We reviewed a range of measures incorporating numeracy and literacy practices from around the world, including those developed for research and survey purposes such as UPskill in Canada (Gyarmati et al., 2014), LSAL in the United States (Reder, 2012) and PIAAC (international) (OECD, 2016b), and for pedagogical and/or career-related purposes, such as Mapping the Learning Journey (Republic of Ireland) (Merrifield & McSkeane, 2005), the Essential Skills Profiles (Canada), the Australian Core Skills Framework (ACSF) and the Occupational Information Network (O*NET) database (U.S.A.). We found that extant measures vary widely, reflecting differences in purpose, scope, context and target audience. A full review of these measures is beyond the scope of this paper; in this section we synthesize our findings and

outline some features of selected measures.

In the research context, various methods have been used to gather data on adults' numeracy practices. For example, Street, Baker, and Tomlin (2005) investigated the meanings and uses of numeracy in school, home and community contexts, using ethnographic-style approaches, including formal and informal interviews and observations. Brown, Yasukawa, and Black (2014) interviewed and observed production workers in three manufacturing companies using an ethnographic approach to understand the complex range of vocational knowledge and social skills that may go unrecognised by policy makers, lobbyists and managers, and even by the workers themselves.

As we have noted above, numeracy may be invisible to those engaged in it and some numeracy activities are not observable. Noss, Hoyles, and Pozzi (2002) addressed this problem in their research on nurses' conceptions of the intensive quantity of drug concentration by devising simulations of "breakdown episodes" in which the nurses' routines were disrupted. They then developed a task-simulation interview schedule to examine the degree of situatedness of the nurses' knowledge and reasoning and to explore the relationship between context and knowledge by manipulating the mathematical relationships in the breakdown episode in ways that varied the discursive distance between the simulation and nursing practice. They found that nurses' conceptions were abstracted from their professional practice but also limited and shaped by their practice.

International surveys have also explored adults' numeracy and literacy practices. For example, Earle (2011) categorizes types of work practices involving numeracy and/or literacy in his analysis of the OECD's Adult Literacy and Lifeskills (ALL) survey as: financial literacy and numeracy (working with invoices and

prices); intensive literacy (reading and writing letters, emails, reports and manuals); and practical literacy and numeracy (reading diagrams and directions, writing directions, measuring and estimating size and weight, and using numbers to keep track of things).

PIAAC is the most comprehensive international survey of adult skills to date and assesses both cognitive skills and practices in the domains covered (OECD, 2016a). According to William Thorn (2014), OECD's PIAAC Manager, these domains were chosen for reasons of efficiency and policy relevance because they are generic, i.e., highly transportable and relevant to a wide range of contexts and situations. In PIAAC cognitive proficiency is scaled through 500 points divided into six levels for numeracy and literacy. PIAAC also provides information on respondents' use of skills at work and in everyday life, their education, linguistic and social backgrounds, participation in adult education and training programs and in the labor market, and other aspects of their well-being. The frequency and types of practices associated with PIAAC domains are targeted in the Background Questionnaire (OECD, 2010) using multiple items applicable to activities in and out of work (OECD, 2016b). Frequency is measured against five categories: never; less than once a month; less than once a week; at least once a week; and every day. The OECD allows access to the anonymized PIAAC dataset with associated tools, providing an opportunity for researchers to explore relationships between practice and cognitive assessments in the PIAAC domains at scale and for specific population groups.

Meanwhile, in the pedagogical/training context, in Canada, the Essential Skills Profiles associated with UPskill measure frequency of use on a six-point scale from "never" to "every day" for nine essential skills used in the workplace, at the level of difficulty required to perform specified jobs successfully. The

essential skills are: reading; document use; writing; numeracy; oral communication; thinking; digital technology; working with others; and continuous learning. Each essential skill contains a list of essential skills-related example tasks, with complexity ratings from Level 1 (basic) to Level 5 (advanced) that vary based on the requirements of the workplace. Essential Skill Function Overviews describe the purpose and/or use of each essential skill (except for Thinking) (ESDC, 2014). The Essential Skills Profiles can be used directly with individuals and can also help build research, standards and curriculum.

The Australian Core Skills Framework (ACSF) describes the core skills of learning, reading, writing, oral communications and numeracy in a five-level framework built on a range of theoretical perspectives, one of which is "a socio-linguistic and socio-constructivist view of core skills as complex social practices embedded in context, and influenced by purpose, audience and contextualised expectations and conventions" (Commonwealth of Australia, 2012, p. 4). The ACSF can be used as a diagnostic tool to assess individuals' literacy and numeracy skills and also as a tool to inform curriculum development and for mapping learning programmes and workplace skill requirements. In addition to skills/knowledge levels it also outlines examples of activities that individuals are able to engage in at each of the five levels. ACSF thus covers both complexity and frequency of practice.

The Essential Skills Profiles and ACSF are unusual in that they include a measure of complexity of numeracy and/or literacy practices; most of the measures we reviewed cover frequency but not complexity. Also, the frequency scales we encountered do not capture intensity of practice. For example, someone working on costings all day and someone else doing so for ten minutes a day would both be reported as doing so 'every day'. We believe frequency,

complexity and intensity of numeracy and literacy practices are all important and should be measured if possible in order to reflect the nature and extent of adults' numeracy and literacy practices.

In summary, our review of existing measures did not reveal a measure that we felt could be taken "off the shelf" for use in the New Zealand context.

Concluding Remarks

So, here is the quandary. Measuring proficiency in numeracy and literacy is relatively straightforward through traditional tests. However, there is likely to be little if any improvement in skill levels from short-term programs (Reder, 2009; Waite et al., 2014). By contrast, practices are where learners are likely to show improvements in both the short and longer-term and engagement in numeracy and literacy practices leads to later proficiency gains.

It is for these reasons that Reder (2013) argues that measures of engagement with literacy and numeracy practices would be a better way of showing continuous improvement during and after engagement with a learning programme. He does not suggest that proficiency measures be dropped, and nor do we, rather that practice measures be developed to complement them.

Our scoping study suggests that a range of matters will need to be considered in further work to develop a measure of adults' numeracy and literacy practices that is valid, reliable, culturally and ethically sound, cost-effective and practicable for use in busy classrooms and workplaces.

In a later ethnographic study we propose to explore methodological issues stemming from our characterization of numeracy and literacy practice, including identifying a methodological framework and unit of analysis and considering whether the LNS distinction between literacy events and literacy

practices could work for numeracy.

Frequency, complexity and intensity of practice will all be considered in our proposed future research and development, as will the possibility of adopting or adapting an existing measure for use in New Zealand. Meanwhile, the importance of assessment in relation to a structured range of complexity of demand is highlighted in research on numeracy for nursing (Coben & Weeks, 2014). Intensity of practice may also emerge as a significant factor in our proposed ethnographic study.

Ethical considerations will be important because of the need to balance the measurement of numeracy and literacy practices with respect for adult learners' privacy. For example, numeracy and literacy feature in adults' engagement in potentially sensitive issues concerning health, personal relationships and money. We envisage that a proposed practices measure would be guided by an ethical framework.

The relationship between a practices measure and numeracy and literacy proficiency, as codified in the New Zealand adult numeracy and literacy infrastructure, will also need to be considered. This is challenging since we know from LSAL (Reder, 2012) that practices and proficiencies are not neatly aligned. We shall also consider how the results of a measure of adults' numeracy and literacy practices might be used expansively and creatively rather than reductively by education and training providers, government and funding bodies, employers and adult learners themselves to support improved learner outcomes (Coben & McCartney, 2016). For such a measure to gain traction it will be important that it is not too onerous for use in busy learning environments.

In summary, it is clear from the research reviewed here that there is a connection between numeracy and literacy practices, attendance in learning programs and learner outcomes. We suggest that knowing the

learner; knowing the demands; and knowing what to do (National Centre of Literacy & Numeracy for Adults, 2011) would be supported by a measure of adults' numeracy and literacy practices. Developing a richer and "technically more sophisticated" measurement methodology in Schuller's (2001) and Reder's (2016) terms, that would acknowledge and validate adult learners' practice gains, inform teaching geared to their circumstances, needs and interests and complement existing proficiency measures, will be a challenging but worthwhile task. ❖

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References

- Balatti, J., Black, S., & Falk, I. (2006). *Reframing adult literacy and numeracy course outcomes: A social capital perspective*. Retrieved from <http://www.ncver.edu.au/>
- Barton, D., & Hamilton, M. (1998). *Local Literacies: Reading and writing in one community*. London: Routledge.
- Bourdieu, P. (1976). *Outline of a Theory of Practice* (R. Nice, Trans.). Cambridge: Cambridge University Press.
- Brown, T., Yasukawa, K., & Black, S. (2014). Seeing and hearing: Examining production workers' literacy and numeracy practices in a context of crisis. *Studies in Continuing Education*, 36(2), 188-200.
- Bunker, B. (2010). *A Summary of International Reports, Research and Case Studies of Digital Literacy: Including implications for New Zealand of adopting a globally-recognised digital literacy standard*. Retrieved from [http://www.iitp.org.nz/files/201001 Digital Literacy Research Report.pdf](http://www.iitp.org.nz/files/201001%20Digital%20Literacy%20Research%20Report.pdf)
- Bynner, J., & Parsons, S. (1998). *Use it or lose it? The impact of time out of work on literacy and numeracy skills*. London: Basic Skills Agency.
- Bynner, J., & Parsons, S. (2009). Insights into basic skills from a UK longitudinal study. In S. Reder & J. Bynner (Eds.), *Tracking adult literacy and numeracy: Findings from longitudinal research* (pp. 27-58). New York and London: Routledge.
- Coben, D. (2000). Mathematics or common sense? Researching invisible mathematics through adults' mathematics life histories. In D. Coben, J. O'Donoghue, & G. E. FitzSimons (Eds.), *Perspectives on adults learning mathematics: Research and practice* (pp. 53-66). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Coben, D. (2006). Social-cultural approach to adult numeracy: Issues for policy and practice. In L. Tett, M. Hamilton, & Y. Hillier (Eds.), *Adult literacy, numeracy and language: Policy, practice and research* (pp. 96-107). London: Open University Press/McGraw Hill.
- Coben, D., Colwell, D., Macrae, S., Boaler, J., Brown, M., & Rhodes, V. (2003). *Adult numeracy: Review of research and related literature*. Retrieved from http://www.nrdc.org.uk/uploads/documents/doc_2802.pdf
- Coben, D., & McCartney, N. (2016). Beyond compliance: Developing a whole organisation approach to embedding literacy and numeracy. In K. Yasukawa & S. Black (Eds.), *Beyond economic interests. Critical perspectives on adult literacy and numeracy in a globalised world* (pp. 119-134). Rotterdam, Boston, Taipei: Sense Publishers.
- Coben, D., & Weeks, K. W. (2014). Meeting the mathematical demands of the safety-critical workplace: Medication dosage calculation problem-solving for nursing. *Educational Studies in Mathematics. Special Issue on Vocational Education and Workplace Training*, 86(2), 253-270. doi:10.1007/s10649-014-9537-3
- Commonwealth of Australia. (2012). *Australian Core Skills Framework*. Retrieved from <http://www.industry.gov.au/skills/ForTrainingProvidersAustralianCoreSkillsFramework/Pages/DownloadtheFramework.aspx>
- Davis, N., Fletcher, J., Brooker, B., Everatt, J., Gillon, G., Mackey, J., & Morrow, D. (2010). *E-learning for adult literacy, language and numeracy: A review of the literature*. Wellington: Ministry of Education.
- Department of Labour. (2010). *Upskilling Partnership Programme evaluation report*. Wellington: Department of Labour
- DfES. (2006). Dyscalculia and dyslexia. *Adult Numeracy Core Curriculum*. Retrieved from http://www.dfes.gov.uk/curriculum_numeracy/tree/number/fractions/accessguidance/5/
- Dixon, S., & Tuya, C. (2010). *Workers with low literacy or numeracy skills: Characteristics, jobs and education and training patterns*. Retrieved from <http://www.dol.govt.nz/publication-view.asp?ID=328>
- Earle, D. (2011). *Literacy and numeracy at work. Skills, education and job tasks*. Retrieved from <http://www.nzliteracyportal.org.nz/imsdirector.php?resid=5197&ruid=1906>
- Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. *Journal of Education and Work*, 14(1), 133-156. doi:10.1080/13639080020028747
- Eraut, M. (2004). Transfer of knowledge between education and workplace settings. In H. Rainbird, H. Fuller, & H. Munro (Eds.), *Workplace learning in context* (pp. 201-221). London: Routledge.
- ESDC. (2014). *Skills definitions and levels of complexity*. Retrieved from <http://www.esdc.gc.ca/eng/jobs/les/definitions/index.shtml>
- Esposito, L., Kebede, B., & Maddox, B. (2012). *The value of literacy practices*. Retrieved from https://www.trentu.ca/ids/documents/Q2_WP59_Esposito_et_al.pdf
- Evans, J. (1999). Building bridges: Reflections on the problem of transfer of learning in mathematics. *Educational Studies in Mathematics*, 39, 23-44.
- Gyarmati, D., Leckie, N., Dowie, M., Palameta, B., Taylor, S.-w. H., Dunn, E., & Hébert, S. (2014). *UPSKILL: A credible test of workplace literacy and essential skills training. Technical Report*. Retrieved from [http://www.clsrn.econ.ubc.ca/workingpapers/CLSRN Working Paper no. 143 - Gyarmati et al.pdf](http://www.clsrn.econ.ubc.ca/workingpapers/CLSRN%20Working%20Paper%20no.%20143%20-%20Gyarmati%20et%20al.pdf)
- Heath, S. B. (1982). What no bedtime story means: Narrative skills at home and school. *Language in Society*, 11(1), 49-76.
- House of Lords Select Committee on Digital Skills. (2015). *Make or break: The UK's digital future. Report of Session 2014-15*. Retrieved from <http://www.publications.parliament.uk/pa/ld201415/ldselect/lddigital/111/111.pdf>

- Hull, G., & Schultz, K. (2001). Literacy and learning out of school: A review of theory and research. *Review of Educational Research*, 71(4), 575-611.
- Hutchings, J., Yates, B., Isaacs, P., Whatman, J., & Bright, N. (2012). *Hei Ara Ako Ki te Oranga: A model for measuring wellbeing outcomes from literacy programmes*. Retrieved from <https://akoaootearoa.ac.nz/download/ng/file/group-5327/the-kaupapa-maori-wellbeing-assessment-model--hei-ara-ako-ki-te-oranga.pdf>
- Keeley, B. (2007). *Human capital*. Paris: OECD.
- Keogh, J., Maguire, T., & O'Donoghue, J. (2012). Mathematics in the workplace - Invisible to whom? In T. Maguire, J. J. Keogh, & J. O'Donoghue (Eds.), *Mathematical eyes: A bridge between adults, the world and mathematics. Proceedings of the 18th International Conference of Adults Learning Mathematics - A Research Forum (ALM)* (pp. 228-237). Dublin: Institute of Technology Tallaght & Adults Learning Mathematics - A Research Forum (ALM).
- Knorr Cetina, K., Schatzki, T. R., & von Savigny, E. (2005). *The practice turn in contemporary theory*. London: Routledge.
- Lane, C. (2013a). *Assessing skills of adult learners in 2011. Profiling skills and learning using the Literacy and Numeracy for Adults Assessment Tool*. Retrieved from http://www.educationcounts.govt.nz/publications/tertiary_education/assessing-skills-of-adult-learners-2011
- Lane, C. (2013b). *Literacy and Numeracy for Adults Assessment Tool 2010-2012: Preliminary ethnic and age group analyses*. Wellington: Ministry of Education.
- Lane, C. (2014). *Literacy and numeracy assessments of adult English language learners*. Wellington: Ministry of Education.
- Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.
- Leont'ev, A. N. (1969/1995). Problems in the psychology of activity. *Journal of Russian and East European Psychology*, 3, 39-53.
- Maddox, B., & Esposito, L. (2011). Sufficiency re-examined: A capabilities perspective on the assessment of functional adult literacy. *Journal of Development Studies*, 47(9), 1315-1331. doi:10.1080/00220388.2010.509788
- Merrifield, J., & McSkeane, L. (2005). *Mapping the learning journey: NALA assessment framework for literacy and numeracy*. Retrieved from <http://www.nala.ie/resources/mapping-learning-journey>
- National Centre of Literacy & Numeracy for Adults. (2011). The '3 Knowings'. Retrieved from <https://www.literacyandnumeracyforadults.com/resources/356174>
- Noss, R., & Hoyles, C. (1996). The visibility of meanings: Modelling the mathematics of banking. *International Journal of Computers in Mathematics Learning*, 1(1), 3-30.
- Noss, R., Hoyles, C., & Pozzi, S. (2002). Abstraction in expertise: A study of nurses' conceptions of concentration. *Journal for Research in Mathematics Education*, 33(3), 204-229.
- OECD. (2010). PIAAC Background Questionnaire. MS version 2.1 d.d. 15-12-2010. Retrieved from [http://www.oecd.org/site/piaac/Background Questionnaire 15DEC10.pdf](http://www.oecd.org/site/piaac/Background%20Questionnaire%2015DEC10.pdf)
- OECD. (2013). *OECD skills outlook 2013: First results from the Survey of Adult Skills*. Retrieved from <http://dx.doi.org/10.1787/9789264204256-en>
- OECD. (2016a). *Skills matter. Further results from the Survey of Adult Skills*. Retrieved from <http://www.oecd.org/skills/skills-matter-9789264258051-en.htm>
- OECD. (2016b). *Technical report of the Survey of Adult Skills (PIAAC) (2nd Edition)*. Retrieved from http://www.oecd.org/skills/piaac/PIAAC_Technical_Report_2nd_Edition_Full_Report.pdf
- OECD, & Statistics Canada. (2000). *Literacy in the information age: Final report of the International Adult Literacy Survey*. Retrieved from <https://www.oecd.org/edu/skills-beyond-school/41529765.pdf>
- Perry, K. (2012). What is literacy? -A critical overview of sociocultural perspectives. *Journal of Language and Literacy Education [Online]*, 8(1), 50-71.
- PIAAC Numeracy Expert Group. (2009). *PIAAC numeracy: A conceptual framework*. Retrieved from <http://www.oecd.org/edu/oecdprogrammeoftheinternationalassessmentofadultcompetenciespiaac-documentation.htm>
- Purcell-Gates, V., Degener, S. C., Jacobson, E., & Soler, M. (2000). *Affecting change in literacy practices of adult learners: Impact of two dimensions of instruction*. Retrieved from <http://www.ncsall.net/fileadmin/resources/research/report17.pdf>
- Purcell-Gates, V., Degener, S. C., Jacobson, E., & Soler, M. (2002). Impact of authentic adult literacy instruction on adult literacy practices. *Reading Research Quarterly*, 37(1), 70-92. doi:10.1598/RRQ.37.1.3
- Reder, S. (1994). Practice-engagement theory: A sociocultural approach to literacy across languages and cultures. In B. Ferdman, R.-M. Weber, & A. G. Ramirez (Eds.), *Literacy across languages and cultures* (pp. 33-73). New York, NY: State University of New York Press.
- Reder, S. (2008). The development of literacy and numeracy in adult life. In S. Reder & J. Bynner (Eds.), *Tracking adult literacy and numeracy: Findings from longitudinal research* (pp. 59-84). New York and London: Routledge.
- Reder, S. (2009). Scaling up and moving in: Connecting social practices views to policies and programs in adult education. *Literacy and Numeracy Studies*, 17(1), 35-50.
- Reder, S. (2011). *Some Thoughts on IALS Measurement Validity, Program Impact, and Logic Models for Policy Development. Think paper*. Paper presented at the Fall Institute 2011 IALS (International Adult Literacy Survey) Its Meaning and Impact for Policy and Practice, Banff, AB. <http://www.centreforliteracy.qc.ca/sites/default/files/Rederthinkpiece.pdf>

- Reder, S. (2012). *Research brief. The Longitudinal Study of Adult Learning: Challenging assumptions*. Retrieved from <http://www.centreforliteracy.qc.ca/publications>
- Reder, S. (2013). Lifelong and life-wide adult literacy development. *Perspectives on Language and Literacy*, 39(2), 18-21.
- Reder, S. (2016). Skill Use: Engagement in reading, writing and numeracy practices. In A. Grotlüschen, D. Mallows, S. Reder, & J. Sabatini (Eds.), *Adults with low proficiency in literacy and numeracy* (pp. 37-59). Paris: OECD Publishing.
- Schatzki, T. R. (2012). A primer on practices: Theory and research. In J. Higgs, R. Barnett, S. Billett, M. Hutchings, & F. Trede (Eds.), *Practice-based education: Perspectives and strategies*. Rotterdam, The Netherlands: Sense Publishers.
- Schuller, T. (2001). The complementary roles of human and social capital. *Canadian Journal of Policy Research*, 1, 89-106.
- Sen, A. (1997). Human capital and human capability. In S. Fukuda-Parr & A. K. Shiva Kumar (Eds.), *Readings in human development: Concepts, measures and policies for a developmental paradigm* (pp. 35-37). Oxford: Oxford University Press.
- Sheehan-Holt, J., & Smith, C. (2000). Does basic skills education affect adults' literacy proficiencies and reading practices? *Reading Research Quarterly*, 35(2), 226-243.
- Stewart, J. (2011). Extending the assessment of literacy as social practice. *Adult Learner: The Irish Journal of Adult and Community Education*, 41-55.
- Sticht, T. G. (2013, 14th Oct., 2013). Key PIAAC Finding: The "Triple Helix" of Literacy Development. *AAACE-NLA list*. Retrieved from <http://lists.literacytent.org/pipermail/aaace-nla/2013/007863.html>
- Street, B. (1984). *Literacy in theory and practice*. Cambridge: Cambridge University Press.
- Street, B. (2000). Literacy events and literacy practices. In K. Jones & M. Martin-Jones (Eds.), *Multilingual literacies: Comparative perspectives on research and practice* (pp. 17-29). Amsterdam: John Benjamins Publishing Company.
- Street, B. (2003). What's "new" in New Literacy Studies? Critical approaches to literacy in theory and practice. *Current Issues in Comparative Education*, 5(2), 77-91.
- Street, B. (Ed.) (1993). *Cross-cultural approaches to literacy*. Cambridge: Cambridge University Press.
- Street, B., Baker, D., & Tomlin, A. (2005). *Navigating numeracies: Home/school numeracy practices*. London: Kluwer.
- Swain, J., Baker, E., Holder, D., Newmarch, B., & Coben, D. (2005). 'Beyond the daily application': Making numeracy teaching meaningful to adult learners.
- TEC. (2016). *Guidelines for using the Literacy and Numeracy for Adults Assessment Tool*. Retrieved from <https://www.tec.govt.nz/Documents/Forms Templates and Guides/Guidelines-for-using-the-Literacy-and-Numeracy-for-Adults-Assessment-Tool.pdf>
- Tett, L., Hamilton, M., & Hillier, Y. (Eds.). (2006). *Adult literacy, Numeracy and language: Policy, practice and research*. New York, NY: Open University Press.
- Thomas, G., & Ward, J. (2010). *Numeracy for adults: Building skills with online learning links*. Retrieved from http://www.educationcounts.govt.nz/publications/tertiary_education/numeracy-for-adults-building-skills-with-online-learning-links/5.-conclusion-and-recommendations
- Thorn, W. (2014). "Riding on unfinished rails" - designing PIAAC. *Elm (European Lifelong Learning Magazine)*.
- Vaughan, K. (2008). *Workplace learning: A literature review. Report prepared for Competenz*. Retrieved from <https://akoaootea.ac.nz/download/ng/file/group-189/n1575-workplace-learning-a-literature-review.pdf>
- Vygotsky, L. S. (1962). *Thought and language* (E. Hanfmann & G. Vakar, Trans.). Cambridge, MA: MIT Press.
- Vygotsky, L. S. (1978). *Mind in society. The development of higher psychological processes* (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Trans.). Cambridge, MA: Harvard University Press.
- Waite, E., Evans, K., & Kersh, N. (2014). The challenge of establishing sustainable workplace 'Skills for Life' provision in the UK: Organisational 'strategies' and individual 'tactics'. *Journal of Education and Work*, 27(2), 199-219. doi:10.1080/13639080.2012.742180

form learning communities. ASolange Farina co-founded the Math Exchange Group (MEG), a teacher collaborative of adult educators in New York City that met to do math and improve math instruction from 1993 until 2012. (Brover, Deagan, & Farina, 2000). Math teachers' circles often provide a space for teachers to work on non-routine problems for which solution paths are not always clear (Fernandes, Koehler and Reiter, 2011; Geddings, White, & Yow, 2015). As a professional development opportunity, these circles encourage content exploration and connected pedagogical conversations (White, Donaldson, Hodge & Ruff, 2013). They have been shown to be effective in providing support for teachers, promoting the use of problem-solving as an approach to teaching mathematics and even changing teachers' views of what mathematics is (Donaldson, Nakamaye, Umland, & White, 2014).

Context

This article discusses the professional development approach used by the New York City Community of Adult Math Instructors (CAMI), of which the co-authors are members. Founded in November 2014, CAMI is a peer-led group of teachers from adult basic education, high school equivalency and college transition. Our teachers come with varied mathematical content knowledge and teaching experience. Some have taught mathematics for years. Other members are relearning mathematics they haven't seen since high school. Very few CAMI members have degrees in mathematics or math education. An average of about eight teachers are present at each meeting and, over the last two years, more than 50 teachers have come to at least one. In general, leadership of the meetings rotates among an informal group of eight members, including the authors.

Problem Posing and Problem Solving

When preparing for meetings and choosing activities to explore, CAMI facilitators are guided by the definition of a mathematical problem as a "task for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific 'correct' solution method" (Hiebert et al., 1997, as cited in Van de Walle, 2003). We seek out problems that have multiple entry points and are accessible to a wide range of learners, while also allowing for extensions into more advanced mathematics.

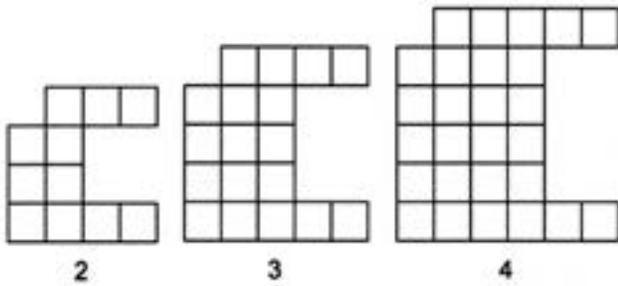
During our meetings, we practice two aspects of teaching and learning mathematics: problem posing and problem solving. We generally start meetings by asking participants to consider a problem and pose questions that come to mind. We then brainstorm more questions in pairs, discuss them as a group and choose questions that we try to answer. We work individually for a while in order develop our own ideas, then work in small groups before presenting solutions to the whole group. The structure of our meetings consists of problem posing, problem solving and presentation of solutions. We base this approach on the teaching of mathematics for understanding through problem solving (e.g., Hiebert et al., 1997), as well as the success of math circles mentioned above.

To illustrate how we use problem posing and problem solving to provide learning opportunities for teachers, we describe a meeting facilitated by Usha Kotelawala. The other authors, along with more CAMI members, participated in the meeting and are quoted below.

A CAMI Meeting

In February 2015 Usha led us through a problem posing activity (Brown & Walter, 2005). She asked us to explore the series of images below (Billings, 2008).

Instead of giving us a specific question to answer, she said: “What do you see? Pose a few questions.”



Individually, we worked for a few minutes to generate questions. This task of coming up with questions, but refraining from working on solutions, proved to be challenging for some. Solange started to make a table of numbers and look for a rule to find the number of squares in any figure. As Usha was walking around to see the questions that teachers were writing, she stopped to talk to Solange.

Usha: “What are you working on?”

Solange: “I want to know the number of squares in the n^{th} figure.”

Usha: “Interesting question. Is that the only question we could pose? For now, let’s just focus on asking questions. We’ll look for answers later.”

Participants continued to generate questions. We then discussed them in pairs and posted our favorites on chart paper at the front of the room. Our questions included the following:

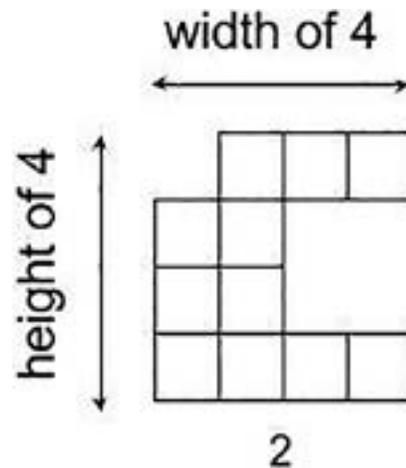
- How many squares are in each figure?*
- What does figure 5 look like? Figure 10? Figure 100?*
- Would figure 5 have an even or odd number of squares? What about figure 10?*

- What is the perimeter and area of each figure?*
- How do the perimeter and area grow for each new figure?*
- What can we learn by exploring the negative space as the figures grow?*
- What is the function for the relationship between the figure number and the number of squares?*

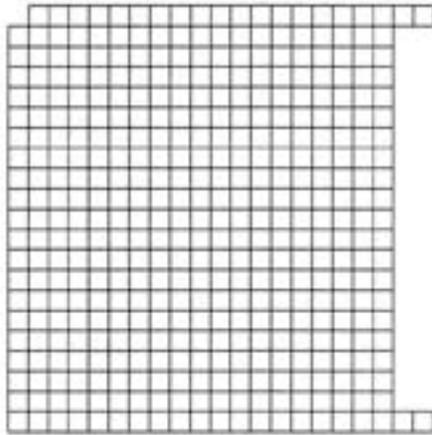
Next, Usha had us reflect on the question-generating activity. This activity allowed us to appreciate how many different kinds of questions can be posed, and, because they came from us, we were invested in answering them. After our discussion Usha had us work together with a partner on a question that interested us. What follows is a description of three presentations shared towards the end of the meeting.

Avril and Mark chose to work on the question: *How would you describe the 19th figure so that someone else could draw it?*

Avril created a chart focusing on the height and width of the figures. She noticed that the height and width of each figure is always two more than the figure number. So, for figure 2, the height and width are both 4.

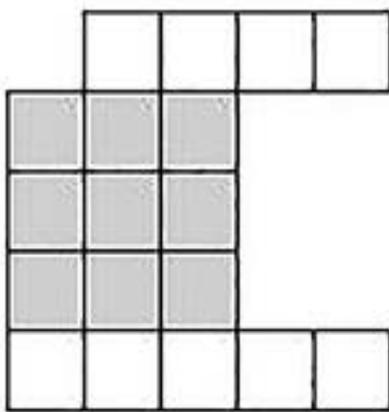


From that, she was able to construct the 19th figure. Extending her method, she knew that the height and width had to be 21.



The 19th Figure

Mark wanted to answer the question without using an equation. He started off by looking at the three given figures and seeing what kinds of patterns he could find. He broke each figure up into three parts: the top row, the bottom row, and the square in the middle.



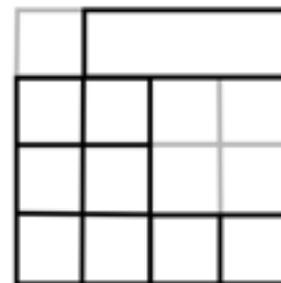
3

Mark's Square in the Middle

He noticed that the top row is always one more than the figure number and that the bottom row is two more than the figure number. He also noticed that the middle was always a square with sides equal to the figure number. Mark used these patterns to write step-by-step directions clear enough for anyone to draw the 19th figure. Avril and Mark both saw the figures differently, but their approaches complemented each other and they were both able to describe the 19th figure.

Solange and Eric were interested in the question: *What can we learn by exploring the negative space as the figures grow?*

Similar to Avril, they first imagined a larger square defined by the height and width: $(n + 2)^2$, where n is the figure number. Then they looked at the squares that were missing from the larger square. They discovered a constant of one missing square in the top left corner of the larger square and a missing rectangle on the right side, which could be described as two times the figure number, or $2n$. From this way of seeing, Solange and Eric developed a rule for finding the number of squares in the n^{th} figure: the larger square $(n + 2)^2$, minus the missing rectangle $(2n)$, minus the constant missing square (1) —or, as an algebraic expression, $(n + 2)^2 - 2n - 1$.



Negative Space in Figure 2

Tyler, Ida, and Alison worked on the questions: *Would figure 5 have an even or odd number of squares? Is there a way to figure out if the number of squares in a given figure will be even or odd?*

These teachers saw that the total squares for the three given figures was alternating odd, even, odd, so they made the generalization that the pattern would continue and the fifth figure would have an even number of squares. They drew the fifth figure and counted its squares to make sure this was true.

Figure	# of Squares
2	11
3	18
4	27
5	38

Tyler made a generalization that allowed him to calculate the number of squares in any figure: $n^2 + 2n + 3$. He then used the expression to explain why an even-numbered figure will always produce an odd number of squares and vice versa. “If the figure number n is odd, n^2 will be odd because the square of an odd number is always odd. Two times n will always be even. Three is always odd. An odd plus an even plus an odd will always be even.”

In her facilitation, Usha asked the three groups to present in a particular order, moving from the concrete to the abstract (Smith & Stein, 2011). We discussed how teachers can use this strategy to orchestrate productive discussions of different problem-solving approaches in their classrooms. We also considered questions that arose from our experience as learners: How can we give our students more time and space to engage with each other’s thinking? How can we help our students adjust to the discomfort of non-routine problems?

Supporting Teachers

At a recent CAMI meeting, members wrote about CAMI and how it has impacted them both as learners and as educators. One member explained that participating in CAMI has enriched her own mathematical learning: “I’ve deepened my mathematical understanding by working on problems that push the boundaries of the math I know, and I’ve learned so much from seeing other teachers’ approaches to problem-solving.” Another member pointed out that CAMI puts him in the position of being a student: “I have that moment where I get anxious and say, ‘She gets it but I don’t get it,’ and it’s that feeling that our students face every day.”

CAMI helps teachers feel supported in an increasingly test-driven adult education landscape in which conceptual understanding is often passed over in favor of teaching procedural skills. One member explained that much of her time now involves monitoring Test of Adult Basic Education (TABE) results, and so her time spent at CAMI meetings is refreshing. As another teacher wrote, “It’s really encouraging to remember that there are so many teachers out there engaged in the same struggle. . . I need constant reminders not to try to cover everything.”

Final Thoughts

The ability to teach math improves as content knowledge grows (Harel, 2008). In order to improve mathematics teaching in adult education, teachers must have positive experiences learning the math they are teaching now, and then reflect on that learning.

CAMI provides a space for teachers to become learners and model the learning environment that we want to create for our students—an environment that few of us had in our own math education. For

many of us, CAMI is the math class we wish we'd had when we were in school and the one we would like to give our students: a place where all voices are heard, where different levels of mathematical experience are welcome, where persistence, curiosity, and elegance are valued in equal measure, and where you formulate your own thinking and learn from the thinking of others. CAMI is a sustainable and replicable model of professional development that impacts us in our roles as teachers *and* learners.

For help starting a math teachers' circle, visit our website—nyccami.org—for math problems, solution methods and discussion notes from our meetings. ❖

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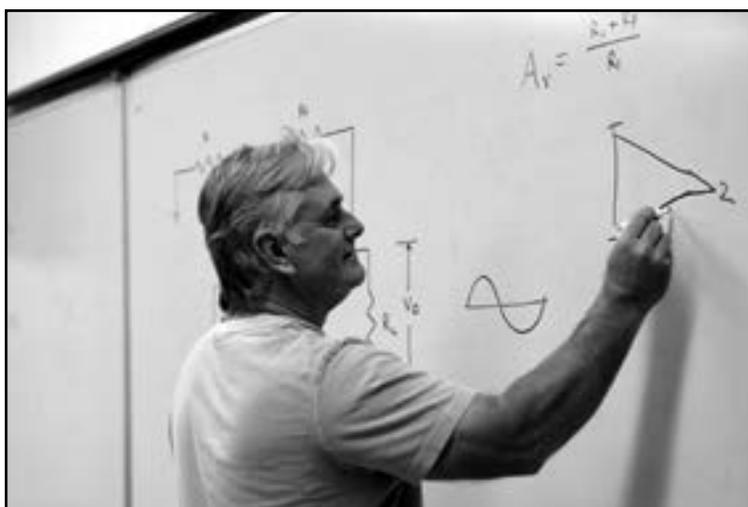
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References

- Billings, E. (2008). Exploring generalizations through growth patterns. In C. E. Greenes & R. Rubenstein (Eds.), *Algebra and algebraic thinking in school mathematics: Seventieth yearbook* (pp. 279–293). Reston, VA: National Council of Teachers of Mathematics.
- Brover, C., Deagan, D., & Farina, S. (2000). The New York City math exchange group: helping teachers change the way they teach mathematics. *Focus on Basics*, 4(1), 1-15.
- Brown, S. I., & Walter, M. I. (2005). *The art of problem posing* (3rd ed.). Mahwah, NJ: Lawrence Erlbaum Associates.
- Donaldson, B., Nakamaye, M., Umland, K., & White, D. (2014). Math teachers' circles: partnerships between mathematicians and teachers. *Notices of the AMS*, 61(11).
- Fernandes, A., Koehler, J., & Reiter, H. (2011). Mathematics teachers circle around problem solving. *Mathematics Teaching in the Middle School*, 17(2), 108-115.
- Geddings, D., White, D., & Yow, J. (2015). SCHEMaTC: The beginnings of a math teachers' circle. *The MathMate*, 8.
- Ginsburg, L. (2011). Adult numeracy demand & provision. A Reader, Four Papers from CAAL's January 2011 Roundtable on Adult Numeracy, 1-27. Council for Advancement of Adult Literacy. Retrieved from <http://www.caalusa.org/Adult.pdf>
- Harel, G. (2008). What is mathematics? A pedagogical answer to a philosophical question. *Proof and other dilemmas: Mathematics and philosophy*, 265-290.
- Smith, M. S., & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions*. Reston, VA: National Council of Teachers of Mathematics.
- Trushkowsky, M. (2015, February 10). Problem-posing with visual patterns. [Blog post]. Retrieved from <http://nyccami.org/problem-posing-with-visual-patterns/>
- Van de Walle, J. A. (2003). Designing and selecting problem-based tasks. In Lester, F. (Ed.) *Teaching mathematics through problem solving prekindergarten - grade 6* (pp. 66-80). Reston, VA: National Council of Teachers of Mathematics.
- White, D., Donaldson, B., Hodge, A., & Ruff, A. (2013). Examining the effects of math teachers' circles on aspects of teachers' mathematical knowledge for teaching. *International Journal for Mathematics Teaching and Learning*. Retrieved from September 26, 2013, at <http://www.cimt.plymouth.ac.uk/journal/>

PRACTITIONER PERSPECTIVE

Stepping Over the Line: Applying the Theories of Adult Learning in a GED Math Class



As a child, I loved watching old Warner Brothers cartoons. One of my favorite character combinations was Bugs Bunny and Yosemite Sam. Invariably, at some point during the cartoon, Bugs Bunny would draw a line in the dirt with his foot and say, “I dare you to step over this line.” Yosemite Sam would step over and reply, “I’m a-steppin.” Bugs Bunny stepped back and drew another line and say, “I dare you to step over THIS line.” Again, Yosemite Sam would step over and say, “I’m a-steppin agin.” This scenario was repeated until Bugs Bunny had maneuvered Yosemite Sam to the edge of a cliff or in front of an oncoming train. Even though Yosemite Sam was in mortal danger, he always took the challenge to step over the line.

In school we are taught to “stay in line” and “color inside the lines.” The connotation of stepping over the line is that you have gone too far in a negative way. But in my recent experiences of being a new adult education graduate student and teacher of a GED Math class, I have found a new meaning for “stepping over the line,” and it is turning out to be a very good thing.

All of my life there has been an invisible line at the front of each math class, whether I had the role of a student or a teacher. The chalkboard of my childhood has been replaced by the whiteboard, but it is always

By

Lisa Bates

the focus of a math class. And the teacher is always standing in front of that board. Going to the board as a student was often terrifying, and as a teacher it is a struggle to get students to solve problems in front of the class. So it is common for teachers to become too comfortable standing at the whiteboard and talking at the students, all the while reinforcing that invisible line between the teacher and the students.

In the Master of Arts in Adult Education program at San Francisco State University, the graduate students are learning Malcom Knowles' theory of andragogy, how adults are more self-directed than children, and how adults bring more experiences to the classroom (Boucoulas & Lawrence, 2010). The graduate students and the professor sit in a circle, discuss and debate in large and small groups, and employ facilitation in learning new topics. The students have an active role in shaping the course of the discussions and readily share their life experiences relevant to the topic.

I was determined to try to incorporate some of these learner-focused teaching techniques and explore ways to adapt them to my GED math learners. My primary goal as a new teacher was to find ways of getting students to participate more in the classroom as a first step towards building community and learning to work together. I reasoned that if students could present their solutions to the class at the whiteboard, then productive student-to-student discussions about math might follow. After a few attempts and failures, I stumbled into ways of incorporating facilitation and collaborative learning into my math class curriculum that draws little lines in the dirt for them to step over. Stepping over these little lines leads to increased knowledge and greater self-confidence in solving math problems as individuals and with other students. And when my students learned to cross that line more freely to the whiteboard, I learned to cross that line into being a student with them. This reflection shows how my

adult learners and I have created a collaborative and supportive learning environment in the classroom where we are all solvers of math problems.

Background

For the 2015-16 school year, the GED morning math classes were on Mondays and Tuesdays from 9 a.m. to Noon. For the GED-Ready math class that I taught, the students had to score at least 145 on the GED-Ready pre-test. Those students who had scored less than 145 were assigned to another class. This score cutoff allowed for an even split of the students and was appropriate given that a passing score on the GED tests was 150. My objective was to cover primarily algebra and geometry so the students could pass the GED Math Test at the end of the semester.

Getting Students to the Whiteboard

I started off with traditional lectures for the first few days in the semester. I spent time getting to know their names, and I asked them to read problems out of the book to gauge their English language skills. On the second day of class, the homework assignments began. As I told the learners, in an ideal world they would go home and do the homework listed on the syllabus before the next class. But in the real world, life often gets in the way of homework. So every morning thereafter we started off the day doing a homework warm-up for 30 to 45 minutes. For students who had done the assigned homework, there were new problems to try in class. For students who did not complete the homework, they had time to work on the homework during the warm-up.

When I first started teaching adult school, I noticed that adult students rarely asked questions, even though it appeared many did not know what to do. So I learned to be specific and I ask, very quietly, little questions of students while I circulate and look over their work: What problem are you working on? Have you tried this one? Did you

check your answer? The first few weeks of class, the students appeared awkward and uncomfortable with me approaching them at their desks. It took them a few days for them to realize that I was trying to catch them solving problems right. I praised them and pointed out what they are doing correctly and asked them to solve another problem like that one. When I saw mistakes, I tried to help them understand where they got derailed. In the early weeks of the semester it is important for me as the teacher to be gentle and supportive.

By the third or fourth week, the students were comfortable with me circulating during the beginning of class. They had heard several of my lectures and had seen how I solve problems on the whiteboard. Many students were comfortable enough to answer questions I posed while I was at the whiteboard. So then I started a new conversation as I circulated in the warm-up period at the beginning of class. After I have reviewed someone's work I might say, "Hey, good job on problem #5! Would you be willing to go to board and show the class how to solve it?" Not many were willing, but by then there were one or two students who agreed to do it. So up to the whiteboard we went, and I stood off to the side if the student needed help. The first student wanted to just write the solution on the board and then sit down. But I encouraged the student to explain the steps. And when the student was done, I told the class to applaud the brave student. From that point forward, the class applauds for every student who presents their solutions at the board.

Once one student goes to the board, there are many others who follow. Around week five, I started the warm-ups by writing the problem numbers that I wanted volunteers to solve in front of the class. The students went up to the whiteboard and put their names next to the problems they wanted to solve. Usually they asked me to check their work first because no one wants to make a mistake in

front of the class. But even on the rare occasion when someone did mess up on the whiteboard, we all applaud at the end. We all understand that mistakes are part of the learning process. I certainly make my fair share of mistakes in front of the class, and I welcome the students' observations on what I did wrong.

I kept track of which students volunteered to go to the board and which students did not. I never forced anyone, but if I noticed someone was not volunteering, then I spent more time with that student during the warm-up. I would find a problem that they solved correctly and ask them specifically to do solve that problem on the board. By week seven, every student had solved a problem in front of the class at least once. And I was no longer standing off to the side in the classroom while a student was presenting a solution on the whiteboard. I was sitting at a desk in the middle of the classroom, surrounded by the other students.

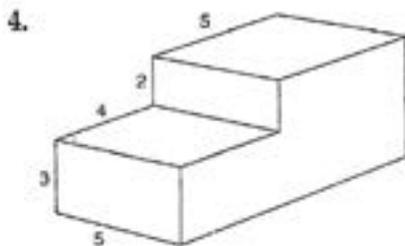
Promoting Collaborative Learning

Of course, there is more to facilitations than getting the students to cross that line and present at the whiteboard. Smith (2010) explains that "in collaborative learning the instructor values and builds upon the knowledge, personal experiences, language, strategies and cultures that the learners bring to learning" (p. 149). As the semester progressed, I added activities that got the students working together and used some of the students' life experiences. For example, to introduce the concept of slope for linear equations, I talked about the OSHA requirement for access ramps, which is "no ramp or walkway shall be inclined more than a slope of one (1) vertical to three (3) horizontal" (OSHA, 1926.451(e)(5)(ii)). Many of my students have had jobs in construction and are familiar with OSHA. One of my students, Frank, offered tips and tricks about ladder safety to the whole class when we started talking about OSHA. For the

slope activity, the students were randomly broken into groups and were tasked with measuring various access ramps around the adult school. After each group collected their data and calculated the slopes, we compared them to the OSHA requirement and discussed the formula for the slope of a line. Working together in small groups and allowing students to bring their life experiences into the classroom helps the students work and think collaboratively.

Learning to work together can produce some exciting learning episodes. Late in the spring 2016 semester, I had assigned the following problem out of the Kaplan GED Test 2015 book (p. 397, problem 4) for the geometry unit:

B. Find the volume of each figure



Eddie had volunteered at the beginning of class to solve this problem on the whiteboard during the homework review. He split this figure into two rectangular prisms by cutting the shape horizontally, so one rectangular prism ($5 \times 5 \times 2$) was on top and another ($5 \times 9 \times 3$) was on the bottom. When Eddie was done presenting his solution, Monica spoke up and said she had done the problem a different way. She cut the shape vertically so there was a rectangular prism in front ($5 \times 4 \times 3$) and a large cube in the back ($5 \times 5 \times 5$). She asked if her way was easier since she was able to apply the formula for the volume of a cube. I asked her to come to the board and present her solution to the problem and the class would discuss the merits of each methodology. Most students agreed that both approaches required two calculations so both approaches were the same amount of work.

Then a third student, Fabian, spoke up and said he thought of a third way to calculate the volume. He wondered if you could think of the figure as one giant rectangular prism ($5 \times 9 \times 5$) minus a smaller rectangular prism ($5 \times 4 \times 2$). I suggested that he come to the whiteboard and try it out. He was reluctant, but Eddie and Monica offered to help him. So I sat in an empty student desk and watched the three of them work out his idea on the whiteboard. I observed the rest of the class taking notes and using their calculators. The entire class was engaged and curious to see if Fabian's third solution was also valid. While it is tempting as the teacher to make suggestions on how to solve a problem, it is much more valuable and engaging for the learners to work out ideas together and be able to discuss the merits of different methods.

I have been amazed at the test results of my students. For fall 2015, there were 10 students in my class for the entire semester. Six students signed up to take the GED Math test, and all six passed. Ten of my 13 spring 2016 students took the GED Math test, and all ten passed the test. While I am proud of the students for passing the test, but I get the greatest satisfaction as teacher by watching the students step over that invisible line to the whiteboard and seeing them realize that they can do math. And we did it together as a little classroom community. It turns out that it is possible for a GED Math class to use facilitations and collaborative learning. I am as surprised as anyone.

Final Thoughts and Suggestions for New GED Math Teachers

- Never give up on a student that appears reluctant to present problems on the whiteboard. One day that student will surprise you by saying yes.
- Be sure to have community classroom agreements in place about being respectful of each other and do not allow teasing. We

can only say positive things in my classroom. The little joke that “Mark” might finally pass the test hurts even if Mark laughs it off. And you might not realize what damage was done until Mark suddenly stops attending.

- Do not underestimate the power of applause after a student has presented to the class. Keep applauding throughout the semester.
- If you stand at the whiteboard and ask if the students have questions, they have no questions. However, if you ask the students to show you the answer on their calculators as you walk around and check, you will get a lot of questions.
- Try something new in your classroom. You might be surprised by the results, but that means you are learning too! ❖

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References

- Boucouvalas, M. & Lawrence, R. L. (2010). Adult learning. In C. E. Kasworm, A. D. Rose, & J. M. Ross-Gordon (Eds.), *Handbook of adult and continuing education* (pp. 35-48). Thousand Oaks, CA: SAGE Publications.
- Kaplan (2015). Kaplan’s GED Test 2015: Strategies, practice and review. New York, NY: Kaplan, Inc.
- Occupational Safety and Health Administration (OSHA). 1926.451(e)(5)(ii). Ramps and walkways. Retrieved from <https://www.osha.gov/pls/oshaweb>.
- Smith, R. O. (2010). Facilitation and design of learning. In C. E. Kasworm, A. D. Rose, & J. M. Ross-Gordon (Eds.), *Handbook of adult and continuing education* (pp. 147-155). Thousand Oaks, CA: SAGE Publications.

*Standards and
Professional
Development*



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Abstract

The purpose of this paper is to describe the professional development that has taken place in conjunction with Ohio adopting the College and Career Readiness (CCR) Standards. The professional development (PD) has changed over time to include not only training on the new standards and lesson plans but training on the concepts defined in the standards. To engage the participants at the PD events, trainings have been developed to include PD over time, hands-on activities with discovery built into the PD, and specific concepts for a discipline. This paper focuses on the changes particularly in the field of mathematics. However, there are PD trainings for reading that have been designed using the same basic principles.

By

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Standards-based education has been a mainstead in Ohio since Equipped for the Future in the mid-1990's, before Common Core State Standards (CCSS) and College and Career Readiness (CCR) Standards for adult education. The first standards for Ohio's adult education program for English Language Arts (ELA) and mathematics were developed by teams of Adult Basic and Literacy Education (ABLE) practitioners. The early ABLE standards were based on testing standards such as TABE and Ohio's K-12 standards. In 2009, they were revised using testing standards, current K-12 standards, and adult education standards from other states.

As Ohio reviewed the CCSS for K-12, noticeable gaps were evident in the Ohio ABLE standards. In 2012, the 129th Ohio General Assembly passed House Bill 153 establishing a remediation-free status¹ and ready for college-level work (Ohio Board of Regents, 2014a). It was evident that more work was needed on the standards to create a stronger alignment with the remediation-free guidelines from H.B.153. In 2014 when Ohio adopted the CCR standards, Ohio ABLE began aligning the standards with the CCR standards for adults and creating a crosswalk to the CCSS for K-12.

A group of trainers from the Professional Development Network (PDN) for Ohio ABLE programs attended institutes offered by the Office of Career, Technical, and Adult Education (OCTAE). At the institute, the participants were introduced to the CCR standards. As part of the training, the trainers unpacked the standards and worked with the content to better understand the nuances of the standards. After the institute, the trainers aligned the Ohio standards with the CCR standards.

In order for local practitioners to know the

content of the newly adapted standards, the PDN created two main documents the *Ohio Board of Regents ABE/ASE Standards*, listing the standards and providing an explanation of the numbering system, and the *Crosswalk of 2014 and 2009 Ohio Standards*, showing the difference between the standards. The numbering system provided a way to connect the previous Ohio ABE/ASE standards to the CCR standards. The numbering system consisted of a content area such as algebra (A) or geometry (G), an NRS Educational Functioning Level (EFL), and a benchmark number. These documents were used to assist the local program providers in "retrofitting" old lesson plans to new lesson plans that were aligned with the newly numbered standards.

Table 1 shows a portion of the crosswalk between the 2014 ABE/ASE benchmarks based on the CCR standards and the 2009 ABE/ASE benchmarks based on the K-12 standards. The 2014 ABE/ASE benchmarks were more detailed and moved higher level skills to lower EFLs as evident from the M.3.11 benchmark (mathematics, level 3, benchmark 11) from the 2009 standards being placed as D.2.3 (data [measurement and data], level 2, benchmark 3) in the 2014 standards.

In implementing standards, it is important to translate them into curriculum and lesson plans (U.S. Department of Education, 2013). In doing this, many opportunities for professional development were developed to ensure that the local program staff know and understand the standards, have lesson plans that address the revised standards, and can provide more engaging lessons.

Professional Development

The research in professional development suggests that one-stop workshops are not as effective as training over time. In a report for the Center for Public Education,

¹ Remediation-free status is a threshold above which a student would not need additional assessment or need consideration for placement into remedial coursework.

five principles for professional development have been identified to create meaningful professional development.

1. The duration of professional development must be significant and ongoing to allow time for teachers to learn a new strategy and grapple with the implementation problem.
2. There must be support for a teacher during the implementation stage that addresses the specific challenges of changing classroom practices.
3. Teachers' initial exposure to a concept should not be passive, but rather should engage teachers through varied approaches so they can participate actively in making sense of a new practice.
4. Modeling has been found to be highly effective in helping teachers understand a new practice.
5. The content presented to teachers shouldn't be generic, but instead specific to the discipline (for middle school and high school teachers) or grade-level (for elementary school teachers). (Gulamhussein, 2013)

These principles suggest that engaging and specific PD over time would be more effective than the one-stop workshop. Changing practice is difficult and it takes time and effort to implement a new practice in the classroom (Guskey, 2002). The philosophy for professional development in Ohio is “to assist ABE program staff in developing the skills and knowledge in order to provide high-quality educational services to assist students in acquiring skills to be successful in postsecondary education/training and employment” (Ohio Board of Regents, 2015). To that end, Ohio has been revising the PD for the teachers to reflect not only the CCSS and CCR standards but also the basic principles of effective PD.

The professional development in Ohio over the last two years has focused on three areas: understanding standards, retrofitting lesson plans,

and building both instructional practices and content knowledge through academies, cohort style trainings, blended learning, and virtual office hours.

Understanding the Standards

The PDN developed a webinar that reviewed the revised ABE/ASE standards and discussed how they related to the previous ABE/ASE standards. The first activity involved discussing the renumbering of the ABE/ASE standards and how the renumbering could be used to provide a quick navigation. In doing this, the trainers pointed out the emphasis that is being placed on the revised ABE/ASE standards to prepare the students to postsecondary education and training. The participants were then led through an exercise that took the newly revised ABE/ASE standards apart to note how they were similar to the previous ABE/ASE standards. The unpacking of the standards assisted the participants to better understand the skills and concepts needed to address the benchmark by focusing on the skills, concepts, contexts, depth of knowledge, and a sample activity. Table 2 shows the template used to unpack the standards. Following the webinar, the teachers were assigned a team and a set of standards to unpack. This training provided the background for the local program staff to be able to “retrofit” lesson plans.

In working with the standards, the biggest concern of teachers is how to possibly cover all the standards. To address this concern, priority benchmarks were identified. The priority benchmarks are a subset of the benchmarks that cover the essential content for the educational functioning level. For example, for Educational Functioning Level 3 in mathematics there were 69 standards for mathematics of which 28 priority benchmarks were identified. Priority benchmarks were identified using the criteria of endurance, leverage, readiness, and cumulative power (Ohio ABE Professional Development Network, 2014b). Priority benchmarks

focus on the skills that are essential to a student's life beyond the classroom. The priority benchmarks can be applicable to other content areas and/or prepare the student for the next level by including other benchmarks. Using priority benchmarks to identify the essential skills did not eliminate any skills to be included in the instructional process. For example, a priority benchmark for reading is (Ohio Professional Development Network, 2014c):

R.1.3. Read with sufficient accuracy and fluency to support comprehension.

- f. Read grade-level text with purpose and understanding.
- g. Read grade-level text orally with accuracy, appropriate rate, and expression on successive readings.
- h. Use context to confirm or self-correct word recognition and understanding, rereading as necessary.

This benchmark can be used for social studies and science as well. An example of a priority benchmark for mathematics is (Ohio Professional Development Network, 2014b):

D.2.5. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

Once the local program staffs were exposed to the CCR standards, the focus turned to translating the standards into lessons for the students. The next focus of the PDN was retrofitting current lesson plans to the new standards.

Retrofitting Lesson Plans

Another concern of teachers is do they have to write new lesson plans since the ABLE programs in Ohio had already existing lesson plans built on the previous ABE/ASE standards. It was important to try to keep the current lesson plans and retrofit them for the revised ABE/ASE standards. Previously used lesson plans were reviewed and kept, if possible, while updating the references to the CCR standards. Besides updating the references, lesson plans were revised to expand the student engagement of the lessons and to include the practices described in the revised ABE/ASE standards. The PDN spent hours working with local program staffs to create lesson plans that were useful and addressed the priority standards. The work on lesson plans was completed on planning time from the local programs.

The hours spent on the PD included virtual office hours, webinars, and PDN staff time reviewing the revised lesson plans. The hours spent were important so that the local program staffs understood the CCR standards and understood how they relate to the previous Ohio ABE/ASE standards. The retrofitted lesson plans that were vetted by the PDN staff are housed in the Teacher Resource Center (TRC) which can be found at www.ohioable.org/TeacherResources. There are over 200 lesson plans ready for use in the ABLE classroom and over 2,000 resources referenced.

An example of the beginning of a pre-retrofitted lesson (J. Franks, personal communication, September 21, 2016) can be seen in Figure 1. The beginning of the retrofitted lesson plan follows in Figure 2 and Figure 3 from Ohio Professional Development Network (2014d).

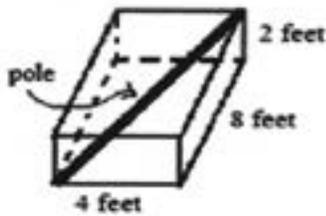
The retrofitted lesson plan includes two additional steps in the process. Step 6 extends the Pythagorean Theorem to three-dimensional items. The first part of Step 6 is shown below (Ohio Professional Development Network, 2014d):

Step 6 – Students have been working with two-dimensional shapes and can now begin

to think three-dimensionally. Share these two problems with students. Use classroom technologies (Smart Board, etc.) available to display the problems and images. Teacher can model a think-aloud working through the problem, then students can work in pairs or individually to solve the second problem.

Problem 1—Rectangular Storage Unit

The dimensions of a rectangular storage unit are 4ft x 8ft x 2ft. What is the length of the longest pole you could fit in the rectangular storage unit?



Calculate the length to the nearest tenth.

Rectangular Storage Unit Answer: 9.2 feet

Step 7 extends the lesson to real world problems (Ohio Professional Development Network, 2014d):

Step 7 - Provide practice using the Pythagorean Theorem in real world problems. There are 2 problems included at the end of the lesson that provide good practice. In addition, investigate Pythagorean Theorem problems on the GED test. Have pairs of students solve each problem. Discuss as a class the clues that “told” them the problem required using the Pythagorean Theorem. Brainstorm with the students situations where right triangles occur, both in two and three dimensions:

- Amount of wire needed to run from the top of a pole to a point 6 feet from the base of the pole
- Straight line distance between locations on roads that are perpendicular to one another

- Diagonal distance of a rectangular picture frame or a TV—how screens are measured
- Length of a ramp when you know the height and linear distance it covers
- Diagonal distance across a park
- Area of a lot
- An octagon shaped deck
- A ladder against the side of a house
- Length of the ramp for a moving truck

The retrofitted lesson example provides the theory behind the Pythagorean Theorem and provides real world application. The lesson extensions add depth to the lesson through discussion, practice, and problem solving.

Building Content Knowledge

Mathematics has been a particularly challenging content area. Only a few of Ohio’s ABE teachers are math majors. It is important that PD helps build their mathematical skills and understanding so that teachers are ready for the classroom.

Ohio began reviewing the existing mathematics PD to see if the trainings are addressing the key shifts in mathematics standards. The three shifts are focus, coherence, and rigor. The focus of the standards is to narrow and deepen the knowledge base to provide a strong foundation for the students. Coherence is a shift to create logical progressions in the content within and across levels. Rigor is a shift to equal measures of conceptual understanding, procedural skill and fluency, and application of mathematics in real world contexts (U.S. Department of Education, 2013).

The CCR standards for mathematics were divided into two parts: Standards for Mathematical Practice and Standards for Mathematics Content. Some of the Standards for the Mathematical Practices are making sense of problems, reasoning abstractly

and quantitatively, constructing viable arguments, modeling mathematics, attending to precision, looking and using structure, and looking for and expressing regularity. These practices help to engage students in doing mathematics. (Ohio Board of Regents, 2014b) The Standards for Mathematics Content suggests that the students need more content to be ready for college and for a career. The revised ABE/ASE standards particularly at the upper levels delve into more algebra by solving for equations and inequalities and using data to understand the relationship of two categories.

The PD that was previously developed needed to be refocused on instructional shifts, mathematical practices, and the expanded content of the standards. Using the current content as the basis for the PD, the instructional shifts and standards for mathematical practice were woven into presentations.

It was clear that in many cases PD offered disjointed content without the rigor of understanding. In many instances, PD was presented in one day with various topics being presented quickly to “get in” the most topics possible. The former PD conducted in Ohio was more typical of a mathematics classroom with the presenter building mathematical knowledge but not necessarily engaging the teachers using the mathematical practices. Professional development that involves the mathematical practices guides teachers in better understanding the mathematical concepts and skills. It is necessary to build the knowledge of mathematical practices of the teachers so the PD in Ohio becomes two-fold: teaching the content that the teachers need to know and using the mathematical practices that will aid in better understanding of the concepts.

An example of the type of PD that was developed is “Math Instruction in Action.” This PD focuses on the basic mathematics needed for college and career preparation. In this training, there is instruction on percent, algebra, geometry, and data. The PD

focuses on participant engagement. For example, the comparison of the volume of two geometric shapes was demonstrated through an activity that showed physically the difference in the volumes. Discussion ensued after the activity which provided a deeper understanding of what affects the volume of a shape. The participants in the training were able to rediscover the volume relationship. This was, to many participants, the first time they enjoyed mathematics. Doing mathematics and not just seeing mathematics takes the teachers into the area of exploration. The techniques learned made math more alive to the 14 teachers who participated. Some of the teachers even said that this is the first time they actually understood the concepts behind the procedures. Several teachers indicated that it was difficult to pick just one item that was most helpful. All the ideas and materials will be used to make the class more engaging. If this would have been a previous training, the content would have been presented with a demonstration of the concepts but there may not have been time for the participants to “get into” the mathematics. The one drawback is that “Math Instruction in Action” is a one-day training. The possibility of expanding it to a multi-day training with follow-up activities to align with the principles of effective PD is being explored.

Another example of PD that Ohio has used is the LINCS training, Adult Numeracy Instruction (ANI). The training deals with many hands-on examples of the mathematical practices in relation to mathematical content that is at an intermediate level. The trainings take place over time as three two-day trainings. To date, three cohorts have completed the training totaling 55 participants. This represents approximately 59% of the 56 ABE programs in Ohio. The acceptance of this intense training was a surprise. The teachers who have gone through the training are telling other teachers that they should attend. Engaging activities are the key for this training. It is important that there be pairs of teachers coming

from the same program so that the teachers have support for the changes that they will need to do in their own classroom. In addition, the local program administrator needs to support the efforts of the teachers in applying the new knowledge and skills to their practice. In supporting the teachers, the administrators can give the teachers “permission” to try things differently. This support will help the teacher to practice their new skills and to try other pedagogical techniques with their students. During the last session of the ANI trainings, the trainers ask about suggestions for what comes next. In addition to the local program support, the teachers asked for more PD like ANI to update the skills that they learned and to extend their mathematical skills. They also asked for ANI-type trainings for other subjects and to have regular meetings to discuss and reinforce their skills.

Once the teachers go back to their classrooms, they are continuing to use ANI-type lessons to engage the students as illustrated from the comments from an informal survey to the participants conducted in April, 2016. One administrator said that more time is spent developing lessons for the entire class to engage in the math as opposed to just presenting to the class and hoping that the lessons reach them. Another administrator said that teachers are using the skills in the classroom, especially using more group work and pairing students of different levels in differentiated learning classes. One teacher stated that the algebra pattern that she uses often is the “why” of positive and negative multiplication results. By following patterns, the students actually see why this is the case, not just memorizing a rule. Another teacher encourages the students to communicate about mathematics by explaining how they got an answer either verbally or in writing. In addition, the students like the new focus of the lessons and are more engaged in mathematics and are coming back for math class.

Ohio ABLE is researching other trainings to expand on the current set of offerings. One such training is the Adults Reaching Algebra Readiness (AR)² through the Adult Numeracy Center at TERC. This training is an extension of the ANI training that has already been successful in Ohio. (AR)² attendees work with linear functions and progresses to system of equations. The focus is always how does mathematics relate to real-life (Adult Numeracy Center at TERC). After just the first session, the teachers are said to have enjoyed the training and are eager to take part in the next session.

An example of ELA training that is being delivered over time is “How to improve Students’ Reading Comprehension by Increasing Their Skills in Alphabets, Vocabulary, and Fluency.” This training is taking place from September 2016 to February 2017 and consists of one face-to-face training, a series of short online courses, and a few live webinars. It is too soon to tell how well it is accepted and if the teachers will accept this type of reading PD.

Conclusion

As the CCR standards have been adopted by Ohio, it is important that the Ohio Department of Higher Education ABLE Program Office and the PDN continue to provide on-going support to ABLE teachers and administrators. The CCR standards require PD to help the teachers understand the standards and to build their expertise in the content area. It is also important to provide the teachers with resources so that they have the materials to use in the classroom. On-going PD takes time to develop. However, the result of developing webinars, hands-on activities, and publications provides a broader scope of training to more local program staff.

This effort to prepare the teachers for the CCR standards has taken and is taking time, expertise, and money. Just like the commitment teachers make to do something differently, the Ohio Department

of Higher Education ABE Program Office had to think differently. Financial resources were adjusted to bring in experts or to develop the expertise within the current state PDN structure. The “one-day and done” training was not as effective as training that takes place over time. This involved having the expertise of the trainers being used for a longer time with the same participants. But the time and effort of the PD staff is paying off. Teachers are using the mathematical practices they experience and are changing their own practice. ❖

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References

- Adult Numeracy Center at TERC. (n.d.) *Adults Reaching Algebra Readiness (AR)2*. [Brochure]. Cambridge, MA. Retrieved from <https://www.terc.edu/display/Projects/Adults+Reaching+Algebra+Readiness+%28AR%292>
- Gulamhussein, A. (2013) *Teaching the teachers: Effective professional development in an era of high stakes accountability*. Alexandria, VA: Center for Public Education. Retrieved from <http://www.centerforpubliceducation.org/Main-Menu/Staffingstudents/Teaching-the-Teachers-Effective-Professional-Development-in-an-Era-of-High-Stakes-Accountability/Teaching-the-Teachers-Full-Report.pdf>
- Guskey, T.R. (2002) Professional development and teacher change. *Teachers and Teaching: Theory and Practice*, 8(3), 381-391.
- Kent State University. (n.d.) Eureka! (Resource file). Retrieved from: http://mercury.educ.kent.edu/database/eureka/documents/LessonPlan_HeyPythagoras_Ohio.pdf
- Ohio Board of Regents. (2015). *Adult Basic and Literacy Education (ABLE) professional development policy and guidelines*. Columbus, Ohio. Retrieved from https://www.ohiohighered.org/sites/ohiohighered.org/files/uploads/able/reference/policies/FY%202016%20PD%20Policy_2.pdf
- Ohio Board of Regents. (2014a). *Ohio Remediation Report*. Columbus, Ohio. Retrieved from https://www.ohiohighered.org/sites/ohiohighered.org/files/uploads/reports/2014-Ohio-Remediation-Report_FINAL.pdf
- Ohio Board of Regents. (2014b). *Ohio Board of Regents ABE/ASE Standards*. Columbus, Ohio. Retrieved from <http://www.ohioable.org/files/Standards/FY15%20CCRs-Ohio%20ABE%20ASE%20Standards.pdf>
- Ohio Professional Development Network. (2014a). *Crosswalk of 2014 and 2009 Ohio standards: Standards for mathematics*. Columbus, Ohio.
- Ohio Professional Development Network. (2014b). *Curriculum alignment: Data*. Columbus, Ohio. Retrieved from <http://trc.ohioable.org/>
- Ohio Professional Development Network. (2014c). *Curriculum alignment: Reading*. Columbus, Ohio. Retrieved from <http://trc.ohioable.org/>
- Ohio Professional Development Network. (2014d). Lessons plans. (Resource file). Retrieved from <http://trc.ohioable.org/>
- U.S. Department of Education, Office of Vocational and Adult Education. (2011). *Standards-in-Action: Innovations for standards-based education*. Washington, DC. Retrieved from http://lincs.ed.gov/publications/pdf/Standards_In_Action.pdf
- U.S. Department of Education, Office of Vocational and Adult Education. (2013). *College and career readiness standards for adult education*. Washington, D.C. Retrieved from <https://www.vrae.org/images/customer-files/CCRStandardsAdultEd.pdf>

Table 1—Crosswalk of 2014 Benchmarks with 2009 Benchmarks

2014 ABE/ASE Benchmark	2009 ABE/ASE Benchmark
D.2.1. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. (2.MD).	
D.2.2. Estimate lengths using units of inches, feet, centimeters, and meters. (2.MD.3)	
D.2.3. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (2.MD.4)	M.3.11 Make, record and interpret measurements of everyday figures.
D.2.4. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. (2.MD.10)	M.2.16 Create and interpret pictographs and bar graphs.

Ohio Professional Development Network (2014a)

Table 2—Unpacking the Standards Template

Standards	Key Knowledge & Skill	Concepts	Particular Context	Cognitive Demand	Sample Activity
State the standard	What skills are students expected to know? (verbs)	What information or ideas should the learner know? (nouns and noun phrases)	What are the circumstances in which students are required to use the skills and concepts?	What is the Depth of Knowledge level for this standard?	What kinds of activities have you used or seen used in the classroom with a good effect?

U.S. Department of Education (2011)

Figure 1—Kent State University, Eureka! Database

HEY, PYTHAGORAS! HELP ME UNDERSTAND YOUR THEOREM!				Student/Class Goal Students may have heard of the Pythagorean Theorem, but are uncertain how to solve these kinds of problems or why they would be important in their everyday lives.	
Outcome (lesson objective) Students will demonstrate their knowledge of the Pythagorean Theorem by creating two problems based on real-life situations.				Time Frame Up to 5 classes	
Standard Use Math to Solve Problems and Communicate				NRS EFL 4-6	
Number Sense	Benchmarks	Geometry & Measurement	Benchmarks	Processes	Benchmarks
Words to numbers connection		Geometric figures	4.7, 5.6, 6.6	Word problems	4.25, 5.25, 6.26
Calculation		Coordinate system		Problem solving strategies	4.26, 5.26, 6.27
Order of operations		Perimeter/area/volume formulas	4.9, 5.8	Solutions analysis	
Compare/order numbers		Graphing two-dimensional figures		Calculator	4.28, 5.28, 6.29
Estimation	5.4	Measurement relationships	4.11, 5.10, 6.10	Mathematical terminology/symbols	4.29, 5.29, 6.30
Exponents/radical expressions	4.6, 6.5	Pythagorean theorem	5.11	Logical progression	
Algebra & Patterns	Benchmarks	Measurement applications		Contextual situations	4.31, 5.31, 6.32
Patterns/sequences		Measurement conversions		Mathematical material	
Equations/expressions		Rounding		Logical terms	
Linear/nonlinear representations		Data Analysis & Probability	Benchmarks	Accuracy/precision	
Graphing		Data interpretation		Real-life applications	4.34, 5.35, 6.36
Linear equations		Data displays construction		Independence/range/fluency	4.35, 5.36, 6.37
Quadratic equations		Central tendency			
		Probabilities			
		Contextual probability			
Materials Graph paper (white), graph paper (colored) Square tiles or 1" paper squares Rulers Colored paper Calculators Pythagoras Vocabulary Self-Inventory Chart Parts of a Right Triangle Handout Pythagorean Triples Handout Pythagoras Learning Objects					
Learner Prior Knowledge Previous experience with square numbers (can be represented by dots in a square array) and square roots (a number when multiplied by itself equals a given number) and the representation of each. Students recognize a right triangle (triangle with a 90° angle). The lesson <i>Quilting Geometry</i> provides students with a background of identifying angles.					
Instructional Activities Step 1 - Each student is given the Pythagoras Vocabulary Self-Inventory chart with a list of words discussed in this lesson. Students identify "I know the word ☺, heard of it ☹ or have no idea ☹," then discuss words already known and words that need to be defined before the lesson. Be sure to review what squaring a number means ($5^2 = 5$ times 5).					

Figure 2—Retrofitted Eureka! Lesson Plan

		Hey, Pythagoras! Help Me Understand Your Theorem!		TEACHER NAME Paula Mullet	PROGRAM NAME			
		Similarity, Right Triangles and Trigonometry		NRS EFL 3-4	TIME FRAME 5-6 classes			
OBR ABE/ASE Standards – Mathematics								
Instructional	Numbers (N)		Algebra (A)		Geometry (G)		Data (D)	
	Numbers and Operation		Operations and Algebraic Thinking		Geometric Shapes and Figures	G.3.4	Measurement and Data	
	The Number System		Expressions and Equations		Congruence		Statistics and Probability	
	Ratios and Proportional Relationships		Functions		Similarity, Right Triangles, and Trigonometry	G.4.8 G.4.9		
	Number and Quantity				Geometric Measurement and Dimensions			
					Modeling with Geometry			
			Mathematical Practices (MP)					
	<input checked="" type="checkbox"/> Make sense of problems and persevere in solving them. (MP.1)				<input checked="" type="checkbox"/> Use appropriate tools strategically. (MP.5)			
	<input checked="" type="checkbox"/> Reason abstractly and quantitatively. (MP.2)				<input checked="" type="checkbox"/> Attend to precision. (MP.6)			
	<input checked="" type="checkbox"/> Construct viable arguments and critique the reasoning of others. (MP.3)				<input checked="" type="checkbox"/> Look for and make use of structure. (MP.7)			
<input checked="" type="checkbox"/> Model with mathematics. (MP.4)				<input checked="" type="checkbox"/> Look for and express regularity in repeated reasoning. (MP.8)				

Figure 3—Retrofitted Eureka! Lesson Plan continued

	
<p>LEARNER OUTCOME(S) Students use the Pythagorean Theorem to solve for the distance between two points and determine how to create a right triangle from two points on a coordinate graph, in real-life situations, and in two- and three-dimensional shapes. (DOK 1-2)</p>	<p>ASSESSMENT TOOLS/METHODS Pythagorean Triples Handout Pythagorean Triples Assessment Two real-life problems submitted to teacher for quiz development Completion of student-generated quiz</p>
<p>LEARNER PRIOR KNOWLEDGE Previous experience with square numbers (can be represented by dots in a square array) and square roots (a number when multiplied by itself equals a given number) and the representation of each. Students recognize a right triangle (triangle with a 90° angle). The lesson Quilting Geometry provides students with a background of identifying angles.</p>	
<p>INSTRUCTIONAL ACTIVITIES Step 1 - Each student is given the Pythagoras Vocabulary Self-Inventory chart with a list of words discussed in this lesson. Students identify "I know the word," "I heard of it," or "I have no idea." "I," then discuss words already known and words that need to be defined before the lesson. Be sure to review what squaring a number means ($5^2 = 5$ times 5).</p> <p>TEACHER NOTE Briefly discuss the Greek mathematician, philosopher and religious leader Pythagoras. Pythagoras, who died about 475 BC, was the first to prove a theorem about right triangles that was known to the Babylonians 1000 years earlier. Check out these web sites for some background information on Pythagoras: Challenging Students to Discover Pythagoras, Pythagorean Theorem and Right Triangle Facts, or Biography of Pythagoras.</p> <p>Step 2 - Identify the parts of a right triangle. The handout <i>Parts of a Right Triangle</i> has an illustration of a right triangle that students can label. Review the mathematical meaning of the Pythagorean Theorem.</p>	<p>RESOURCES Pythagoras Vocabulary Self-Inventory Chart</p> <p>Challenging Students to Discover Pythagoras https://www.teachingchannel.org/videos/pythagorean-theorem-lesson Pythagorean Theorem and Right Triangle Facts http://www.math.com/school/subject3/lessons/S3U3L4GL.html Biography of Pythagoras http://www.groups.dcs.st-and.ac.uk/~history/Mathematicians/Pythagoras.html</p> <p><i>Parts of a Right Triangle Handout</i></p>

What's an Adult Numeracy Teacher to Teach? Negotiating the Complexity of Adult Numeracy Instruction

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In light of U.S. adults' dismal performance on the recent PIAAC numeracy assessment, we certainly have to improve math and numeracy instruction in all sectors of our society. Sixty percent of American adults scored lower than Level 3 of 5 numeracy levels, with 20% scoring at Level 1 and 9% scoring below Level 1 (based on a nationally representative sample of 5,010 adults). A larger percentage of the adults scored at the lowest levels on the numeracy assessment than on the lowest levels of the literacy assessment, indicating a particularly urgent need to address numeracy. Twenty-four countries participated in the PIAAC assessment program; the US numeracy scores were third from the bottom (better than only Italy and Spain) and significantly below the international average. (OECD, 2013).

Other data confirm that mathematics is a particular challenge for adults. One study found that 59% of entering community college students were required to take at least one developmental math course based on standardized placement tests; only 33% completed their assigned sequence of courses. In contrast, 33% of entering students were required to take at least one developmental reading course and 46% completed their assigned sequence (Bailey,

Jeong, & Cho, 2009). Historically, the annual reports from the high school equivalency test companies have revealed higher failure rates for their mathematics assessments than for the other tests in the battery.

The data described above address two sides of the challenge facing adult educators. The PIAAC assessment focused on numeracy and addressed adults' ability to manage and respond to situations in everyday life, work, society and further study, by identifying, acting upon, interpreting, evaluating, and communicating embedded mathematical information and ideas (PIAAC Numeracy Expert Group, 2009). In the community college study, the standardized placement tests, such as the Accuplacer, typically present mathematics questions that resemble traditional school-like tests, focusing on symbolic manipulations without meaningful contexts.

While "mathematics" is seen as an organized body of content that is school-based, abstract and decontextualized, "numeracy" may be less clearly defined. Most definitions of numeracy emphasize "situatedness" that encompasses the mathematical components and personal dispositions, reasoning and practices that individuals purposefully use in their personal, social and work-related activities.

Robert Orrill contrasts numeracy and mathematics, explaining “unlike mathematics, numeracy does not so much lead upward in an ascending pursuit of abstraction as it moves outward toward an ever richer engagement with life’s diverse contexts and situations” (2001, p. xviii).

As adult educators, we have long been aware that the K-12 educational system has not been effective for many people. While improving adults’ mathematics and numeracy must be a high priority, educators who strive to address their learners’ educational gaps and needs are finding themselves in an environment of competing priorities and expectations that can be expressed as mathematics numeracy education. What should guide the content of math/numeracy instruction in the midst of competing priorities emanating from the US Department of Education, federal and state legislation, research, adult education programs or agencies, and from our learners?

Standards and Regulations

College and Career Readiness Standards

In 2011, the U.S. Department of Education released the College and Career Readiness Standards (CCRS) and has been supporting their implementation with technical assistance projects. These Standards are derived from the Common Core State Standards for K-12, and primarily include standards from grades 2 through 8. To me, the most significant aspect of the CCRS is the emphasis on developing deep understanding of mathematics through using representations such as number lines and drawings and by emphasizing the importance of students explaining and justifying their reasoning as well as identifying patterns and repeated reasoning. The CCRS explicitly state that conceptual understanding, procedural skills and

fluency, and application should all be pursued with equal intensity (p.44), although the standards provide little guidance on “application” strategies, methods, or approaches.

Since the release of the CCRS, many programs have encouraged teachers to examine the standards and revise lessons so that they are aligned with the standards. One hopes that instruction is truly reoriented from a focus on mastering procedures and meaningless routines (e.g., “keep-change-flip” as a procedural mnemonic for dividing fractions) to explorations of how dividing fractions is related to and different from dividing whole numbers, how multiplying and dividing fractions are related, and when/where the operations are used. While it is easy to make superficial changes to lesson plans by including identifying numbers of CCRS Mathematical Practices and Content Standards, actually developing deep conceptual understanding during lessons is quite a bit more difficult.

Indeed, some adult education teachers may not feel prepared to help learners develop conceptual understanding because they may not have that understanding themselves. Many adult educators teach all subjects to their learners and feel more confident and competent developing literacy skills. Their own educational backgrounds and work histories may not have focused on mathematics. A national survey in 1994 found that while more than 80% of adult students receive some math instruction, less than 5% of adult education teachers are certified to teach math (Gal & Schuh, 1994). While this survey was completed more than 20 years ago, I would doubt the findings would be much different today.

Workforce Innovation and Opportunity Act

The 2014 Workforce Innovation and Opportunity

Act (WIOA) promotes workforce preparation and postsecondary education as the “core purpose” of federally-funded ABE/ASE programs (U.S. Department of Education, 2014). Many of the reporting requirements focus on employment status and post-secondary educational targets. Thus program and agency funding will likely be tied to these priorities.

Adult education programs are expected to develop partnerships with workplaces and industry organizations. To me, this assumes that the adult learners should be preparing for entry or advancement in work settings and/or preparing to enter a work-related certificate program. These expectations imply that the content of adult math/numeracy instruction should be informed by the needs of and preparation for particular employment sectors.

Of course, different jobs and industries require particular bodies of mathematical content knowledge and may have particular ways of applying that knowledge. For example, construction workers likely need to have an understanding of geometry including shapes and angles, fractions, and proportional reasoning so they will be able to read and interpret blueprints, measure accurately, and use their materials efficiently.

Some twenty-first century production workers may be working on computers, requiring complex work practices using computerized modeling software and a different type of work-team environment. Yasukawa, Brown, & Black (2013) report on a group of production workers who manipulate and adjust three-dimensional images of hearing aid shells to fit individual's ears while making sure the electronic components can be placed appropriately. Like the construction workers, these workers are also using their spatial skills, but their

job preparation required the development of “techno-numeracy skills.” They require fewer traditional numeracy skills than what might be “learned” in an educational setting where math instruction is guided by the College and Career Readiness Standards.

Additional research has explored the mathematical practices and understanding of bank workers (Kent, Noss, Guile, Hoyles, & Bakker, 2007; Noss & Hoyles, 1996), nurses (Hoyles, Noss, & Pozzi, 2001; Marks, Hodgen, Coben, & Bretscher, 2015), and workers in a variety of other sectors (FitzSimons, 2013). Findings consistently show that mathematical activity is deeply embedded within the work and is often practiced using procedures that are idiosyncratic to the workplace and that are often learned informally from coworkers. Indeed, Keogh, Maguire and O'Donoghue (2014) state, “mastery of routine mathematics alone was a poor indicator of a person's ability to ‘do the job’ (p. 85).

These descriptions of the rather narrow math content that is used in various workplaces and how that math is used, lead to the conclusion that if adult education is to propel people into the workplace or toward workplace training programs, then the math that is taught should be closely aligned to the particular employment sector. It follows, then, that adult numeracy teachers will need to be intimately familiar with the local industry sectors and the mathematical content and numeracy practices involved in their work.

To me, these two simultaneous emphases of implementing the CCRS and preparation for the local workforce present a powerful challenge for adult math/numeracy instructors. On the one hand, the instructors are expected to teach all mathematical content areas (numbers and operations, algebra, geometry and data and statistics), emphasizing meaning and understanding. On the other hand,

instruction must also prepare learners for the numeracy demands of particular workplaces.

Indeed, the construction employer might expect employees will be able to order fractions, at least those in everyday construction work (maybe halves, thirds, fourths, eighths, sixteenths and thirty-seconds, but maybe not care about fifths, sevenths, ninths or seventeenths), but would the hiring manager at the bank care about that knowledge? That hiring manager might appreciate an employee's understanding of exponents and compound interest, but not necessarily an employee's understanding of the impact on the volume of a cylinder of increasing the radius.

And yet, the CCRS and WIOA are not the only drivers in adult education.

Other Factors

High School Equivalency (HSE) Tests

Currently, there are three High School Equivalency Tests being offered across the country. As mentioned above, many adult students continue to struggle to pass the mathematics tests. The content of the HSE math tests is said to be aligned with the current standards, with a strong emphasis on algebra.

While WIOA does not seem to prioritize attainment of a HSE credential as a primary goal, a high school diploma or HSE credential is generally required to gain access to a work-related certificate program, acceptance to higher education, or even as a minimum educational requirement for many jobs. I once spoke to a 50-year-old laid off truck driver who enrolled in an adult education program because he could not get hired to do the same work he had done successfully for over 30 years without having a high school diploma.

At initial entry to many adult education

programs, learners are often led through goal setting exercises. They are urged to think beyond the HSE test – to look at successfully passing the test as only one step on a longer life and job journey. Regardless, adult numeracy instructors do need to prepare their students for the HSE math tests, even if that is perceived to be only an initial hurdle.

Some educators choose to prepare learners for the HSE test by spending hours practicing sample questions from test prep books. While familiarity with the types of test questions can be helpful, learners do not come away with much understanding of the mathematical content and thus are less likely to be successful on the HSE assessment or any additional assessments used for entry or placement purposes for certification programs or further education. Better preparation for the HSE might be to delve deeply into the mathematical concepts and procedures, particularly addressing algebra.

Adult Learners' Personal Priorities

Finally, adult learners return to study mathematics for their own reasons. Sometimes, they are primarily motivated by a desire to attain or improve employment prospects or are focused on acquiring a HSE credential. But often, their priorities are more personal – they want to be able to help their children with their homework, they want to master content that they were unable to master at an earlier time in their lives and thereby “prove something to themselves,” or they want to “graduate” from high school so as to be a role model for their children or family members (Coben et al., 2007; Jackson & Ginsburg, 2008). These priorities should also be considered when adult numeracy instructors make decisions about what content to teach.

So, how can or should we, adult math/numeracy educators, make instructional decisions given the

myriad of competing demands from federal and state Departments of Education; federal legislation connected to funding sources and agency reporting requirements; HSE test specifications; and adult learners' own motivations to study mathematics? And, of course we are compelled to do this under conditions in which we all know that adult learners have limited time and resources that they can devote to their own formal education. Ultimately, we have to ask "To whom are we accountable?" Can we provide the educational experiences for our learners so that they will have the mathematical tools and problem solving skills as well as the formal credentials they need to identify and accomplish their own dreams and goals and meet the expectations of program funders, test makers, and other external entities? What support and resources do we, as adult numeracy educators, require so we will make informed decisions and implement them most effectively for the benefit of our diverse learners? ❖

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References

- Bailey, T., Jeong, D.W., & Cho, S-W. (2009). Referral, enrollment and completion in Developmental Education sequences in community colleges. (CCRC Working Paper #15). New York, NY: Columbia University, Teachers College, Community College Research Center.
- Coben, D., Brown, M., Rhodes, V., Swain, J. Ananiadou, K., Brown, P., ... Storey, V. (2007). *Effective teaching and learning: Numeracy*. London: National Research and Development Centre.
- FitzSimons, G. (2013). Doing Mathematics in the Workplace. A Brief Review of Selected Literature. *Adults Learning Mathematics: An International Journal*, 8(1), 7-19.
- Gal, I., & Schuh, A. (1994). Who counts in adult literacy programs? A national survey of numeracy education (Report Tr94-09). Philadelphia: University of Pennsylvania, National Center on Adult Literacy. Retrieved from <http://files.eric.ed.gov/fulltext/ED377382.pdf>
- Hoyles, C., Noss, R., & Pozzi, S. (2001). Proportional reasoning in nursing practice. *Journal for Research in Mathematics Education*, 31(1), 4-27.
- Jackson, K., & Ginsburg, L. (2008). Algebra for all? The meanings that mothers assign to participation in an algebra class. *Adults Learning Mathematics — An International Journal*, 3(2a), 10-28.
- Kent, P., Noss, R., Guile, D., Hoyles, C., & Bakker, A. (2007). Characterizing the use of mathematical knowledge in boundary-crossing situations at work. *Mind, Culture, and Activity*, 14(1-2), 64-82.
- Keogh, J.J., Maguire, T., & O'Donoghue, J. (2014). A workplace contextualization of mathematics: Measuring workplace context complexity. *Adults Learning Mathematics: An International Journal*, 9(1), 85-99.
- OECD (2013). *Survey of Adult Skills: First Report*. Retrieved from <https://www.oecd.org/skills/piaac/Country%20note%20-%20United%20States.pdf>
- Orrill, R. (2001). Mathematics, numeracy, and democracy. In L.A. Steen (Ed.). *Mathematics and democracy* (pp. xiii-xix). Woodrow Wilson National Fellowship Foundation.
- PIAAC Numeracy Expert Group (2009). *PIAAC Numeracy: A conceptual framework*. OECD Education Working Papers. No. 35. OECD Publishing. Retrieved from http://www.oecd-ilibrary.org/education/piaac-numeracy-a-conceptual-framework_220337421165
- U.S. Department of Education (2014). *The Workforce Innovation and Opportunity Act Overview of Title II: Adult Education and Literacy*. Retrieved from <http://www2.ed.gov/about/offices/list/ovae/pi/AdultEd/wioa-overview.pdf>
- Yasukawa, K., Brown, T., & Black, S. (2013). Production workers' literacy and numeracy practices: Using cultural historical activity theory (CHAT) as an analytical tool. *Journal of Vocational Education and Training*, 65(3), 369-384.

Where to Focus so Students Become College and Career Ready

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What should guide the content of math and numeracy instruction with seemingly competing priorities from federal and state legislation, research, adult education program demands, and from our learners? Do teachers focus on workplace skills, college-readiness skills, the College and Career Readiness Standards for Adult Education (CCR), or high stakes assessments? Considering the math levels of so many of our adult learners, I believe that teachers can focus on all of these competing demands at the same time, but only if they teach their students how to reason mathematically, and ensure that they have a solid conceptual foundation so that they can apply that knowledge and reasoning to any new situation that arises.

Unfortunately, teachers feel the need to swiftly get students to meet goals and expectations, whether it is passing the test, mastering a CCR Standard, or preparing for college or training. Teachers may hear that 'trig' is now on some of the high stakes assessments and suddenly they feel a need to teach their students some basic trigonometry procedures. Or, they notice that the CCR Math Standards at Level E include factoring of quadratic expressions, so they feel that they need to teach their students procedures associated with that content. Unfortunately, too

many teachers feel like they don't have the time to give students the foundation that would allow their students to actually understand what is being taught. They may teach students procedures and tricks, hoping that they will retain those procedures long enough to at least pass the test.

However, without foundational understanding, students rarely remember those procedures. How many times have teachers shown students how to add fractions with unlike denominators, only to discover a few weeks later that students have already forgotten the procedure? Or, they watched students apply the procedure for adding fractions when faced with a proportion problem? As a result, teachers reteach the same procedures over and over again, rarely successfully getting their students to understand when to use those procedures. According to Givvin, Stigler, and Thompson (2011):

Without conceptual supports and without a strong rote memory, the rules, procedures, and notations they had been taught started to degrade and get buggy over time. The process was exacerbated by an ever-increasing collection of disconnected facts to remember. With time, those facts became less accurately applied and even more disconnected from the problem solving situations in which

they might have been used. The product of this series of events is a group of students whose concepts have atrophied and whose knowledge of rules and procedures has degraded. They also show a troubling lack of the disposition to figure things out, and very poor skills for doing so when they try. This leads them to call haphazardly upon procedures (or parts of procedures) and leaves them unbothered by inconsistencies in their solutions. (p. 5)

Although Givvin et al. (2011) were referring to students supposedly ready for developmental education classes, most practitioners would say that the description would readily apply to the vast majority of learners in their own adult education classes.

Teachers think that they don't have the time to spend on conceptual understanding of core concepts. But, perhaps teachers need to reconsider what it means to be college and career ready, and what it means to have a core set of skills that allow learners to meet the demands of both academic and life priorities. The National Center on Education and the Economy (NCEE) asked: *What does it really mean to be college and work ready?* They conducted a two-and-a-half year study to try to answer that question. What they discovered is most of the math that is required of students before beginning college courses and the math that most enables students to be successful in college courses is not high school mathematics, but middle school mathematics. Ratio, proportion, expressions and simple equations, and arithmetic were especially important (NCEE, 2013). In other words, if we could help our students develop strong math skills at levels A through C/D in the CCR, they would be well-prepared to tackle college level classes or even ready to succeed in training required at the workplace.

And, according to *Redefining College Readiness*, a report published by the Educational Policy Improvement Center (Conley, 2007), college success requires key cognitive strategies such as analysis, interpretation, precision and accuracy, problem solving, and reasoning. Students who are ready for college possess more than a formulaic understanding of mathematics. They are able to apply conceptual understandings in order to extract a problem from a context, use mathematics to solve the problem, and then interpret the solution back into the context. While these skills are specifically called out for college readiness, I doubt anyone would argue that they are not also critical for dealing with life issues and work situations. In other words, students need to have strong reasoning and problem-solving skills for success, not just know a bunch of procedures.

Perhaps it is not only that teachers claim that they don't have enough time to prepare their students for multiple goals. Maybe there is another issue involved. Certainly it is not a lack of commitment or caring on the part of our adult education teachers. However, so few have learned math conceptually themselves. It is rare to find a practitioner who not only understands the procedures herself, but also knows how to teach that understanding.

Compounding the problem, says Ma (1999), in the United States, it is widely accepted that elementary mathematics is "basic," superficial, and commonly understood... Elementary mathematics is not superficial at all, and anyone who teaches it has to study it hard in order to understand it in a comprehensive way" (p. 146). If teachers think that elementary and middle school math is "basic," it might explain why so little time is taken to ensure that our students (including our adult learners) really do understand what those elementary principles are.

Taking into account a widespread attitude that the "lower level" math is easy (and therefore able to

be reviewed quickly) and the number of teachers with limited knowledge of how to teach math, adult education is hard-pressed to get students to reach any of the conflicting goals and expectations. However, if students had a strong foundation of math concepts, they would be able to transfer their understandings to the workplace, to tests, and to situations involving math in their lives. If they are only taught procedures, how will they ever know when to use them on the job or in a college class or on a test?

Teachers should ensure their students have a strong conceptual foundation before launching into “higher level” math. Too often, the students have incomplete mastery of “middle school” math and could use more than just a quick review. Teachers would do well to adopt strategies to strengthen foundational knowledge, such as probing number sense or asking students to predict what an answer will be BEFORE having them jump to the formal calculation. Students who learn to question the logic of their answers are more likely to intuit that the solution to a problem like $5/6 + 1/2$ must be larger than 1, since $5/6$ is greater than $1/2$. In contrast, a student relying on an incorrectly internalized fraction addition procedure might arrive at an answer of $6/8$ —an answer that would stand out as incorrect to a student with solid number sense.

What are some ways that teachers can begin to teach more conceptually so that their students can at least develop some solid skills at the elementary and middle school level while developing mathematical reasoning at the same time? Here is a sampling of ideas, which are based loosely on the CCR Math Standards:

- Introduce the concept of the benchmark $\frac{1}{2}$ (along with its equivalents .50 and 50%) to students who are at Level A. Knowing $\frac{1}{2}$ is more important than knowing how to do long division.
- Build on those benchmarks very slowly—still at Level A, ensuring that students really do understand. Have them apply those benchmarks to data where they can begin to reason critically about simple data representations (beginning with two categories and building to three or four).
- Teach estimation strategies early on and expect students to use them in everything they do, not just when it’s covered in a particular chapter of the book.
- Begin to introduce the concept of proportional reasoning early on by having students build in/out tables as a way to work on basic multiplication facts. Encourage them to discover patterns in the multiplication tables so they begin to see the relationship between different rows in the tables (i.e., 2 to 3, 4 to 6, etc.). Build on the in/out tables by having them begin to create graphs of those patterns. This anticipates the introduction of linear functions (which is middle school level math).
- At Level A, introduce the basic properties of operations and hammer those ideas home as students move from whole numbers to fractions. After all, the properties work just as well for fractions as they do for whole numbers and abstract algebra.
- At all levels, teach conceptually by helping students visualize what is happening. Often seeing a visual representation helps students to understand (and trust) a particular procedure that they have been taught.
- At all levels, ask students to reason about their answers. Don’t listen for the right answer and then move on. Ask students—whether their answer is right or wrong—to explain their thinking. This will go a long way in helping them develop critical reasoning skills.

If teachers focused on these ideas, they would be preparing their students for all of the goals and expectations placed on them. Teachers who not just teach procedures but also conceptual understanding give students a foundation from which to add new knowledge. Students can make connections among math content. For example, if students can understand the idea that the area is the product of two numbers, then they can use that same understanding to visualize why two fractions multiplied together have a product less than either of the two fractions; and, they can use that same understanding to multiply binomials. Teachers then don't have to teach mnemonics such as FOIL (first, outer, inner, last) because students can apply knowledge built from whole numbers. Even if teachers do not get to the topic of binomials, students can use their foundational knowledge for new, more advanced topics. After all, math is not a series of disconnected topics but rather a coherent body of knowledge made up of interconnected concepts.

Teachers who contextualize number and operation sense are already giving students opportunities to practice using skills in the workplace, community, and home. Students might not know specific content needed for work, but students with the ability to apply their learning in different contexts will be able to use their math skills and reasoning in different work environments.

Those teachers who struggle to see how to teach math more conceptually and with more real-life applications should use any available opportunities to further develop their own teaching skills. Most of us were taught in very decontextualized, procedural-based classrooms. Therefore, teachers tend to teach as they were taught. And, if a teacher has spent most of her career in education, it is sometimes difficult to find examples of how to contextualize math lessons.

What can a teacher do to begin her own journey

of learning how to teach math to ensure students succeed? Here are some questions teachers should ask themselves:

- *Do I try to provide students with real-life examples but can only seem to think of my own personal experiences in the kitchen? Seek out opportunities to engage in workplace education and training environments. Minimally, it might be helpful for a teacher to observe how math is applied in an I-BEST or other integrated education and training initiative. Or, even better, to seize the challenge to co-teach in such an environment. Also, take time to have conversations with students about the kinds of jobs they now hold (or would like to) and where they use math within those contexts.*
- *Do I tend to look for short workshops that will provide me with tricks on how to teach 'higher level math'? If so, look for professional development offerings that include opportunities to explore math content in more depth to allow you as a teacher to become a learner for a while. Those quick tricks do not help the teacher, much less her students, develop understanding. And, without the understanding, math will continue to be a set of disconnected procedures to memorize.*
- *Do I use the same scope and sequence that I used five years ago? Ten? Is it based on how I learned math – whole numbers first, then fractions operations (all of them in one unit), decimals, ratios, geometry, etc.? If so, then you might want to explore the College and Career Readiness Standards in more detail to see how the domains (such as operations and algebraic thinking, measurement and data, geometry, and number sense) are integrated. Algebraic*

thinking begins at level A and fractions also begin at Level A (under geometry where students visualize benchmark fractions). When designing lessons in number and operation, think about how and where someone might use a skill.

The real question we need to consider is not how to address competing priorities but rather how to help teachers develop their own understanding so that they can prepare their students for success, no matter what the goal or expectation. ❖

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References

- Common Core State Standards Initiative (2016). *Key shifts in mathematics*. Retrieved from <http://www.corestandards.org/other-resources/key-shifts-in-mathematics/>
- Conley, D.T. (2007). *Redefining College Readiness, Volume 3*. Educational Policy Improvement Center. Eugene, OR. Retrieved from <http://evergreen.edu/washingtoncenter/docs/conleycollegereadiness.pdf>
- Givvin, K., Stigler, J., & Thompson, B. (2011). What community college developmental mathematics students understand about mathematics, Part II: The interviews. *MathAMATYC Educator*, 2(3), 4-18.
- Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates.
- National Center on Education and the Economy. (2013) *What does it really mean to be college and work ready? The mathematics required of first year community college students*. Retrieved from <http://ncee.org/college-and-work-ready/>

Time Well Spent: Making Choices and Setting Priorities in Adult Numeracy Instruction

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In her Forum piece, *What's an Adult Numeracy Teacher to Teach? Negotiating the Complexity of Adult Numeracy Instruction*, Lynda Ginsburg sets the stage of the current problem (poor numeracy levels in American adults) and the bevy of standards, legislation, and new exams that have recently been developed to address it. Ginsburg also highlights some of the ways in which these different “remedies” also compete with one another, as well as with the priorities of adult students. In the midst of all these demands, how does a teacher decide what to teach?

Context

Every program is unique, but there are also challenges that tend to be common across the field. I have been teaching in adult education for almost seven years in a non-profit setting in Dorchester, Massachusetts, in programs that serve women experiencing homelessness or at risk of homelessness. I came to adult education having studied math in college, which seems to be rather uncommon.¹

My background in math is significant because the College and Career Readiness Standards for Adult Education (CCRS) make high demands on a

teacher's “decompressed” mathematical knowledge. In other words, teachers must know the material they are teaching more deeply and conceptually than would be expected of the average adult (Ball, Thames, & Phelps, 2008). In adult education, most teachers are asked or required to teach a variety of subjects, and teachers may not have specialized knowledge or training in math. This is a critical ingredient – not necessarily to have studied advanced mathematics at a collegiate level, but to have spent some time unpacking core mathematical concepts and developing one's own mathematical practices. Teachers who don't see themselves in the Standards for Mathematical Practice (outlined in CCRS, p. 48-50), who haven't themselves developed those ways of thinking and doing mathematics, will need to pursue professional development opportunities to deepen their own understanding. There is no way around it: procedural knowledge is not enough to teach people to understand math conceptually. Although I came into adult education with a solid understanding of advanced mathematics, I still needed to spend time in professional development exploring the foundations of math more deeply, more visually,

¹Ginsburg cites the research of Gal and Schuh (1994) who found that less than 5% of adult numeracy teachers had a certification to teach mathematics. I imagine that the percentage would still be low today even if we widened the category to include mathematics degrees without a teaching focus.

and in a more connected way. This exploration into the foundational concepts of mathematics not only helped my teaching, but even changed the way I appreciate math.

The student body I work with faces challenges common to many adult students. For a variety of reasons, my students' time in school is limited, and time on task can be challenging as well. The impact of trauma on learning is very apparent, and often limits time on task even further. With the majority of my students entering our program with math skills between 3-6 GLE, I have to be very strategic when prioritizing my curriculum. I never know how much time I will have to work with someone, and I want to make sure it is time well spent.

Curriculum Considerations

Over time, I've developed my own informal criteria for making decisions about what to include, what to emphasize, and what to leave out. To me, prioritizing means not just teaching certain topics first and others later, but also making a conscious decision not to teach certain topics presented in the CCRS, WIOA, and/or the High School Equivalency (HSE) exams. I believe this is in the spirit of the "Focus" shift in the CCRS math standards, which says that "instructors need both to narrow significantly and to deepen the manner in which they teach mathematics" (p. 44).²

The first thing I consider when making curriculum choices is *Does it meet my students where they are?* This question is critical in order to teach math conceptually and in a coherent way. With a student body coming to me with upper elementary level

math skills and profound gaps in basic number and operation sense, there is no reason for me to include high school curriculum until I have a cohort that is ready for that level. If I insisted on teaching them material at a much higher level than they are currently at, they would have no option but to learn the math procedurally, because they would be missing all the deep, concrete foundations that abstract reasoning is built on.

My second consideration is *Can they use it now?* When I begin my planning for each unit, I start by thinking of at least one way in which my students can go home and use the math they are learning to enhance their life right now. For example, learning about data can help them better understand the news; learning about ratios can help them find the best deal; learning how number lines work can help them read the gauges on their car and oven. If I can't come up with anything, that unit gets shelved until I can (prioritizing!).

As an adult, we really do learn much more thoroughly and deeply the things we can use immediately. Relevance to our lives taps into our buy-in and intrinsic motivation to learn,³ and it gives us an opportunity to continue to practice and deepen our skills for more permanent mastery. Showing people that math is useful is one of the most important reasons I teach.

My last consideration is similar: *Can they use it again soon?* This usually means I am looking for an opportunity in which my students will be called on to use their mathematical learning again substantially within a few months, or, at the outer limits, a year. This opportunity often comes in a subsequent math

² Curiously, I encounter many teachers who feel that they still don't have permission to leave anything out. It would be interesting to look into where this comes from: administrative pressure? The standards themselves?

³ For an interesting discussion of relevance in teaching, see Roberson, R. (2013) Helping Students Find Relevance. American Psychological Association. Available at <http://www.apa.org/ed/precollege/ptn/2013/09/students-relevance.aspx>.

unit that builds on previous knowledge, or a unit in another subject which will ask them to apply these skills. For example, I follow up a unit on benchmark fractions with a unit about data, which provides numerous opportunities for students to apply their knowledge of basic benchmarks. A map unit in Social Studies might provide the perfect use for skills around ratio and scale. With students in our work readiness program, students interested in health careers work on ratios and measurement; students working towards administrative careers learn to write basic functions in Excel. My hope is that within the next six months, they will have the opportunity to make use of at least some of the skills they learned in their internships, cementing their learning and helping them in their careers.

In addition to enhancing the current lives of my students, this last criterion allows me to consider ways in which the math content can *expand* their life in some way. Perhaps interest rates were not relevant to a particular student before because she was never involved in her own finances or paying taxes, but now she can venture to look into these things. Maybe someone has always relied on other people for home improvement projects, but now has the understanding of measurement to buy and install shelving herself.⁴

“To whom are we accountable?”

Are my students, when they leave me, “College and Career Ready?” This is the phrase that everyone

is struggling to define and to measure. The CCRS has mathematics content through level E. Have my students mastered all that content by the time they leave me? Of course not. In any case, most of the level E content standards definitely fail my own criteria on all three points.

The HSE tests of 2017 define and measure college and career readiness by the achievement of a specific score: for example, HiSET considers a subject test score of 15 to indicate college readiness (ETS, 2016). I would estimate that most of my graduates, who tended to pass the 2016 HiSET with a range of 9-11,⁵ are probably about 1 year worth of instruction short of reaching that mark. To be ready to enter college level math classes, my graduates need (and are encouraged) to participate in a college transition program to boost their skills.⁶

So if the measures of college and career readiness proposed by these different sources aren’t always attainable, how do I move my students as far down the continuum as I can? As a math teacher, one of the gems from the CCRS that is within my humble grasp is to help my students develop the eight Standards for Mathematical Practice, which can be integrated at every level of math content. Not only do these habits of mind make students better at mathematical reasoning, but also at problem solving and effective communication. I do see evidence of improvement in almost all of my students in their mathematical habits of mind. These are skills that will be useful to them both now and soon, transferrable to many

⁴ Based on my second and third criteria, I consider my math curriculum to be contextualized, although the applications are not always related to a specific career. For most of my students, those type of specialized applications will not happen soon enough for me to teach them now.

⁵ Given the importance of receiving their HSE to my students, all of my criteria would go out the window if my curriculum didn’t also prepare them to pass the test. Historically, and consistently, students who are with me long enough do pass the math subtest of the HiSET.

⁶ In Massachusetts, Adult Basic Education programs like mine were recently allowed to continue serving adult students after they received their HSE, but the idea of using adult education programs for this purpose does not seem to have caught on yet with students—not with my students, at least.

types of academic work and work place enigmas.

As a math teacher, I can't fix all the gaps and systemic difficulties that make "College and Career Ready" both vague and seemingly unattainable. Instead, I focus on getting my students "College and Career Ready-er." I don't know how much time or consistency I will get with any given student, and so I prioritize everything I teach to make sure it has relevance on several levels, and if it doesn't, well... it can wait.

Ginsburg concluded her article with the question, "To whom are we accountable?" I see myself as accountable to all of the competing agencies and priorities she mentioned, *but only to the extent to which I can realistically fulfill each of those demands.* In addition, of course, I have my own agenda as a teacher, which is to alleviate some of the math anxiety I encounter in almost all of my students, and to help them see math as understandable, useful, and desirable knowledge to attain. In this way, I am hoping to move the needle, even a little bit, on beliefs about what mathematics is all about and who it is for. These are beliefs that can get passed on to the next generation and possibly effect some long term change.

Support for Adult Numeracy Instructors

Ginsburg also poses a question about what type of support adult numeracy teachers need in order to prioritize and carry out effective instruction. In my experience, there are three critical pillars of support that have contributed to my ability to navigate and teach among all these competing priorities: access to high quality professional development, a robust set of curricular materials to work from, and administrative buy-in and support.

Although I already had a mathematics degree when I began teaching adult numeracy, the additional training I received through the Massachusetts

System for Adult Basic Education Support (SABES) has greatly enhanced my effectiveness as a math instructor. Like most adult numeracy instructors, teaching math conceptually bears little resemblance to my own math education, and the training has been invaluable.

Secondly, I discovered excellent curriculum materials designed for adult students which focus on both conceptual development and relevant adult applications. (I use the series *EMPower*, developed by the Adult Numeracy Center at TERC in Cambridge, MA [Schmitt et al., 2005, 2015].) Developing curriculum is a very specialized skill set, and not all teachers have the time or expertise to create all their material from scratch. For me, having a model of good lessons in the *EMPower* series helped me to even begin to envision what a rigorous math classroom would look like. This is an important missing piece for many adult numeracy teachers: if their own mathematics education was traditional and procedural, and they have not ever seen what teaching math conceptually would even look like, how can they be expected to begin creating lessons from that void?

Lastly, I attribute my modest successes (and my enjoyment!) of my math classroom to the support I have received from my supervisors and administration to teach math this way. It is not easy for many administrators (and teachers) to let go of some of the perceived benefits of traditional, procedurally taught math: on paper, it appears much more efficient; it doesn't require a major overhaul of teacher training and curriculum; and it does fit the expectations that most adults have of a math classroom. I have been fortunate to have directors who believed me when I said students needed to work on foundational skills, who supported my suggested timelines for topics, and who backed me up when students were frustrated

that I wouldn't just tell them the steps.

If we want to move the field of adult numeracy forward to meet these new demands, we need to meet the field where it is: in need of professional development, in need of high quality curriculum, in need of administrative and bureaucratic support. Just like our students, our programs and teachers need relevant goals with realistic timelines. Change is a long process, and we need to be both moving in the right direction and taking care to sustain everyone involved for the long journey ahead. ❖

Melissa Braaten has been teaching adult numeracy for the past six years in Dorchester, Massachusetts, to adults with all levels of math ability and confidence. She is also a consultant for the SABES (System of Adult Basic Education Support) PD Center for Mathematics and Adult Numeracy, and has worked on developing and facilitating online and in-person trainings and workshops for adult numeracy practitioners.

References

- Ball, D., Thames, M., & Phelps, G. (2008) Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407.
- ETS (2016). HiSET Scoring and Credentials. Retrieved from hiset.ets.org/tcs-ae/about/scoring-credential
- Gal, I., & Schuh, A. (1994). Who counts in adult literacy programs? A national survey of numeracy education (Report Tr94-09). Philadelphia: University of Pennsylvania, National Center on Adult Literacy. <http://files.eric.ed.gov/fulltext/ED377382.pdf>
- Schmitt, M., Steinbeck, M., Curry, D., Donovan, T., Merson, M. & TERC. (2005) *EMPower: Extending mathematical power*. Emeryville, CA: Key Curriculum Press.
- Schmitt, M., Steinbeck, M., Curry, D., Donovan, T., Merson, M. & TERC. (2015) *EMPower Plus: Extending mathematical power*. Columbus, OH: McGraw-Hill Education.

RESOURCE REVIEW



*Reviewed by
Nicole A. Taylor*

Spelman College

No Small Lives: Handbook of North American Early Women Adult Educators, 1925-1950

By Susan Imel and Gretchen T. Bersch (Editors)

2014; Information Age Publishing, Charlotte, NC
304 pages, softcover, \$45.99

In the field of adult education, there is a lack of attention paid to the scholarship of women who helped to shape the field. Imel and Bersch address this gap by highlighting the significant contributions of 26 North American women who were active in the field of adult education between the years of 1925 and 1950. *No Small Lives* serves as a catalyst to restore women to their rightful place in history, by narrating the varied and important accomplishments of some of the women who contributed substantially to the growth and development of the field of adult education.

The first section of the book, “Historical Background,” provides a historical lens to set the tone for the remainder of the book. In these sections, the reader learns about the roles and participation of women in adult education in its earliest years and plausible suggestions for how history may be rewritten to be more inclusive of women. For example, the inception of the American Association for Adult Education is described along with the different types of publications written by women signifying their commitment to the vision of adult education.

The second section, “Profiles of 26 Women,” brings to life the stories of 26 diverse women who at one point may have been considered invisible, yet were active in some capacity in adult education between 1925 and 1950. The women included editors, writers, and practitioners whose work brought legitimacy to the field of adult education through

advocacy, involvement, planning and leadership. One may consider this second section to be the “heart” of the book as it provides space for 26 unique stories of the North American women adult educators. Moreover, within this section, the reader has no choice but to be inspired as concrete evidence is brought forth that demonstrates how active and involved women were in adult education during the referenced time period. This section of the book brings clear visibility to women in the field of adult education, where they historically may have been considered invisible. These include woman whose stories may not have been told publically, yet are part of an undoubtedly necessary legacy that contributes to the field’s growth and development over time. This section of the book is filled with stories of women who were leaders and central figures in a fight for power, equality, and growth. In the next few paragraphs, I will proceed to provide a few illustrative examples of a few of the highlighted female leaders.

Nannie Helen Burroughs, a prominent Black educator, church leader, and social activist, is among the women highlighted. Burroughs’ notable efforts included founding the National Training School for Women and Girls in Washington, D.C. (1909), as a national model school for the teaching of African American women. Furthermore, active in the political arena, Burroughs did extensive work in the area of woman’s suffrage by forming organizations and writing articles key to empowering these woman. Today, her legacy continues, as she was a strong influence among women in general and in the school that still exists today.

As an early pioneer in adult education, Mary L. Ely contributed tirelessly as editor to the first professional journal in the field of adult and continuing education, the *Journal of Adult Education*. During a time where adult education was a new and unchartered field, Ely led the call for scholarly work to authors from around the United States. A key player in the adult education movement, Ely edited other important works such as *Adult Education in Action* and *Handbook for Adult Education*.

Remaining true to her culture, Maria L. Hernandez had a mission to assist the Mexican community through the founding of several community organizations. Focused on the disparities in the education of Mexican American children, Hernandez engaged in advocacy to bring awareness to issues of equality. In addition to the fight for equal education, Hernandez was also involved in movements related to women’s rights. A trailblazer in her own right, Hernandez was a pioneer who fought for educational rights an activist, wife, mother, and grandmother.

The last section of the book, “Conclusion,” skillfully provides emphasis on a question that brings the reader to the current context for adult education

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No Small Lives provides a pathway to the voices of the women as it highlights their contributions towards contemporary practices in adult education.

and whether or not “things have changed.” This is an important question to allude to in the conclusion as it seemingly bridges the gap between the past, the present, and the future of adult education. Intuitively, as the book considers what has been done in the field it is a natural next step to consider any implications or future considerations, as the need for the field of adult literacy continues to be present.

Based on the specialized nature of this book, the primary audience would seemingly be those who are intimately involved in the history and academics of adult education such as adult education professors and college students. It is clear that this book could be used as a learning resource in higher education courses as it focuses on the history and sociology of adult education and women’s contributions in adult education. Furthermore, as the book can be seen as an examination of women’s roles in the early twentieth century, it seems as though another audience for this book could be others in higher education in related fields such as anthropology, psychology, sociology, and/or women’s studies. Moreover, it could also be useful to those focusing on more specific topics such as gender and race studies, prejudice, marginalization, power, leadership and policy making.

No Small Lives: Handbook of North American Early Women Adult Educators, 1925-1950 is unique in its approach to reach back to a place in history that has been, what many consider overlooked and under analyzed, to inform the adult education practices of today. Even though there was diversity amongst the 26 women in nationality, ethnicity, educational background, family status, and time period, there was also great commonality found in that they were advocates, leaders, and scholars in the field of adult education. *No Small Lives: Handbook of North American Early Women Adult Educators, 1925-1950* provides a pathway to the voices of the women as it highlights their contributions towards contemporary practices in adult education.

Nicole A. Taylor is an Assistant Professor at Spelman College in the Education Department. At Spelman, she works with pre-service teachers and instructs in the areas of Educational Psychology and Reading. Her research interests include the development of emergent literacy skills in children whose parents are struggling adult readers and the home literacy environment. She earned her doctorate degree from Georgia State University in Educational Psychology with a concentration in language and literacy. She is also involved with the Prime Time Family Literacy program as a Humanities Scholar.



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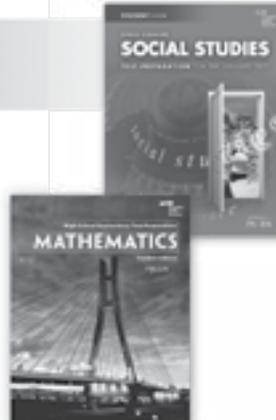
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WEB SCAN



*Reviewed by
David J. Rosen*

Newsome Associates

Numeracy and Math Websites

This Web Scan column has benefited from the numeracy and mathematics expertise of adult education teachers and teacher educators Connie Rivera and Brooke Istas. Many thanks to them for their help.



How Big Is the World's Largest Deliverable Pizza? (Area of Rectangles)

1. Robert Kaplinsky's Search Engine

<http://robertkaplinsky.com/prbl-search-engine/>

If you are looking for a problem-based, real-life numeracy lesson for your students, Connie Rivera suggests this free search engine to look through great math and numeracy websites such as the Mathematics Assessment Project's MAP Mathshell, Wouldyourathermath.com, 3-Acts Lessons, Mathalicious.com, and many others. Here's a sample problem from the website: "How much is One Third of a Cup of Butter?" When I selected it, I was taken to a numeracy lesson plan, in which students look at a butter label, and cut a 4 oz stick of butter together. The lesson plan includes nine Common Core standards addressed by the lesson, such as "CCSS 3.NF.3b Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model."

2. Open Middle, Challenging Math Problems Worth Solving

<http://www.openmiddle.com/>

Connie says this website is one of her top recommendations. She says, “These problems look basic and don’t use a lot of words, but they make you think deeply about what you are doing. A few of these can replace a whole ‘worksheet.’ They are also searchable by a Common Core standard number.” According to the website’s description open middle problems have a “closed beginning” meaning that they all start with the same initial problem, a “closed end” meaning that they all end with the same answer, and an “open middle” meaning that there are multiple ways to approach and ultimately solve the problem. These open middle problems “require a higher depth of knowledge than most problems that assess procedural and conceptual understanding. They support the Common Core State Standards and provide students with opportunities for discussing their thinking.

3. Math Solutions

<http://mathsolutions.com/free-resources/>

Connie recommends this Marilyn Burns website and also for teachers’ professional growth, Burns’ blog, Connie says, “She writes so clearly and makes me see, sentence by sentence, how to be a better teacher.”



4. You Cubed at Stanford University

<https://www.youcubed.org>

Connie says that at *you cubed* you can read recent research about growth mindset and the value of mistakes, visual learning, and the connection between math anxiety and the way we may be teaching math facts. You can also watch video examples and search for tasks to use with students.” Connie adds, “Jo Boaler rocks!”

5. Desmos

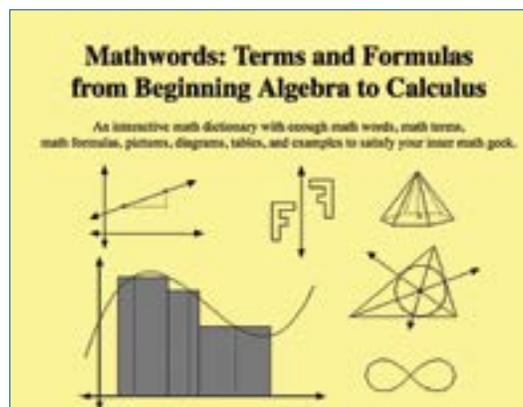
<https://www.desmos.com/>

Brooke Iestas thinks this free math web site is a great way to explore graphs and how small transformations can have a great impact on the way a graph looks. There are also math activities, so if you need some examples for how to incorporate a lesson, or perhaps increase your own personal math knowledge, Brooke says this site is easy to navigate with lots of information.

6. Mathwords

<http://www.mathwords.com>

Students often get confused by math vocabulary. Brooke says, “This is an interactive math dictionary with enough math words, terms, formulas, pictures, diagrams, tables, and examples to help learners begin to speak mathenesel!”



7. GCF Learn Free

<http://www.gcflearnfree.org/topics/math/>

I have recommended GCF Learn Free before in this column, but with a reminder from Brooke, this time I call your attention to its numeracy and math topics such as addition, subtraction, multiplication, division, fractions, decimals, and algebra. Brooke mentions that it also provides tutorials and math interactives.



8. National Library of Virtual Manipulatives

<http://nlvm.usu.edu/en/nav/vlibrary.html>

Brooke suggests this website “to help develop a conceptual understanding of all kinds of mathematical concepts. The Algebra has several wonderful manipulatives to help learners with factoring by giving them a visual, and helping them to manipulate it to create understanding.” Although not free, its \$29.95 price tag for a Windows or Mac computer may fit many teachers’ budgets.

9. Absurd Math

<http://www.learningwave.com/abmath/>

Brooke also recommends *Absurd Math*, an interactive mathematical problem solving game series. She says, “It is a great way to engage the learner and help develop deeper understanding of mathematical concepts.”

Comment from a Web Scan Reader: Dorothea Steinke, an adult numeracy/mathematics teacher in Lafayette, CO wrote me about a course that was included in the futurelearn.com website that I had featured in the Summer, 2016 Web Scan column. She cautioned that U.S. students who try the Numeracy Skills for Employability and the Workplace course, <https://www.futurelearn.com/courses/numeracy-skills> could run into difficulties because “Europe uses the point, where we use commas, and vice versa. So 23,247 in England means 23.247 in the United States.” She added that “the pencil-and-paper processes for multiplication and division may be different from those used in the United States.”

David J. Rosen is an education consultant in the areas of adult education, technology, and blended learning.



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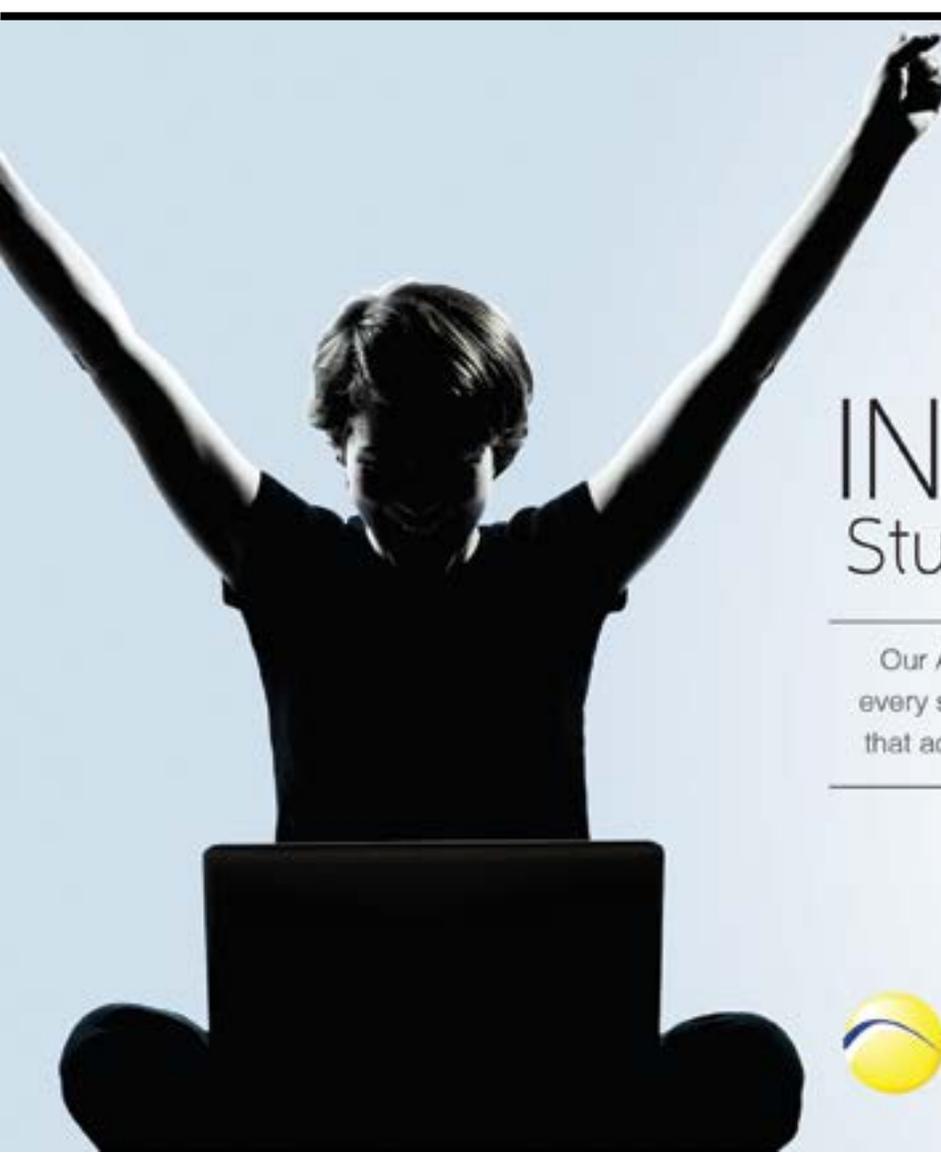
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